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Ministry of
Education

Mowat Block
Queen's Park
Toronto, Ontario
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GRADE 8 GEOMETRY

NOTES FOR TEACHERS

CONTENTS

(Notes for G4 and G5 are under preparation and will be distributed later)

The resource notes in this module are related to the Grade 8 Geometry Strand for Intermediate Division Mathematics 1977, Draft Copy. They are intended for use by teachers and board curriculum committees as they plan the mathematics program for their schools.

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Ministry of Education Ontario

GRADE 8 GEOMETRY

SECTION 1: ACCURATE CONSTRUCTIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Page 12; perpendicular, parallel, vertical, horizontal
	Ed PJ Div:	Page 76
	Gr 7:	N 8a; A 1b; A 3b; G 1; G 2; G 3; G 4bc; G 5b; G 6d
PRESENT	Gr 8:	A 2; N 2b; N 6a; N 7; G 1; G 2ac; G 3ad; G 4bc; G 5b
FUTURE	Gr 9 Gen:	N 3a; N 6; G 1; G 2e; G 3; G 4b
	Gr 9 Adv:	N 5; G 1a-d; G 2; G 3; G 4a
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	Gr 10 Adv:	G 1a-d; G 3; G 4

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	Gr 10 Gen:	N 7a; G 1; G 2bc
	Gr 10 Adv:	G 1a-d; G 3; G 4

a) Constructing congruent segments and angles, perpendicular lines, parallel lines, perpendicular bisector, angle bisector; tests for parallel and perpendicular lines

Students in Grade 8 should already be familiar with the mathematical concepts associated with congruent segments and angles, perpendicular and parallel lines, perpendicular bisector, and angle bisector. All of these have been referred to, either explicitly or implicitly, in The Formative Years and in the program for Grade 7. It is also likely that the students have had experiences with some of the construction techniques associated with these concepts -- at least at an informal level using a variety of instruments and materials. The notes for this topic in Grade 8 are intended as an extension of the suggestions made in the notes for 7G 2a).

The purpose of this section is to ensure that the students are familiar with a variety of techniques for constructing figures accurately, using the materials and/or instruments that are available to them at the time. By learning a variety of construction techniques, the student will consolidate his/her understanding of the concepts associated with perpendicularity, parallelism, bisectors, and congruent figures.

Perpendicular and Parallel Lines

The notes for 7G 2a), pages 2 to 7, indicate a number of methods for testing two lines for perpendicularity or parallelism, and for constructing such lines. Methods using tracing paper for doing this are described in detail in these notes, since

they are simple to use and they complement the methods using tracing paper for the study of translation-, rotation-, and flip-images, and of symmetry.

See the notes for 7G 1b) page 3. Other ways of testing for perpendicularity include:

- using a ruler with identical scales on both sides;
- using grid paper (or rectangular dot paper) to test whether it is possible to superimpose the perpendicular pattern on the lines;
- drawing a line ℓ perpendicular to one of the two lines, then testing whether ℓ is perpendicular to the other line;
- folding a sheet of tracing paper twice to produce perpendicular creases, then testing whether it is possible to superimpose the pattern on the lines.

Are there other ways?

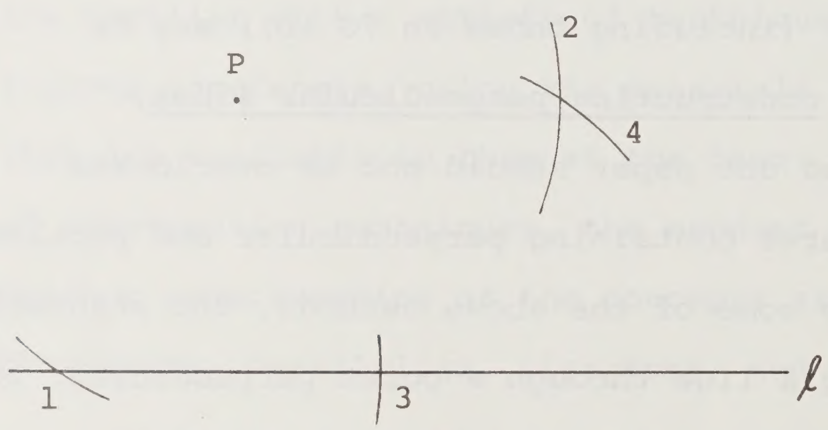
Any of the above tests (including those in 7G 1b)) may be used as the basis for constructing perpendicular lines. In particular, grid and dot paper should not be overlooked when constructing figures containing perpendicular and parallel lines. In addition to some of the above methods, the standard method of constructing a line through a point perpendicular to a given line using a compass and straight edge should be learned. It is assumed that the students have had some prior experiences drawing circles and arcs of circles with a compass (8N 3d, 8N 7, 7G 1abc). If this is not so, then time should be taken to instruct the students on suitable techniques of using a compass. These are not discussed here, because they have been in the curriculum for many years.

See the notes for 7G lb, page 5. Other ways of testing for parallel lines include:

- drawing a transversal to the lines, then comparing the sizes of alternate angles (or corresponding angles), using tracing paper or a protractor;
- placing one half of a parallel rule along one line, then trying to fit the other half on the other line;
- superimposing grid paper (or dot paper) with a line in its pattern along one of the lines, then observing the position of the other line relative to the pattern.

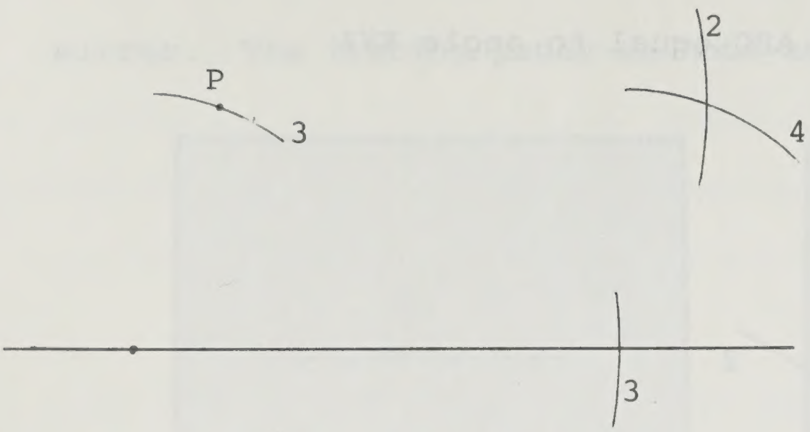
Are there other ways?

Any of the above tests (including those in 7G lb) may be used as the basis for constructing parallel lines. The standard method of constructing a line through a point P parallel to a given line using compass and ruler should be learned. One technique for this is illustrated below.



Draw the required line through P and the intersection of arcs 2 and 4.

When the same radius is used for each of the arcs, the four points are vertices of a rhombus. This technique is called the rhombus method. Students should also learn the 'parallelogram method', as illustrated below.



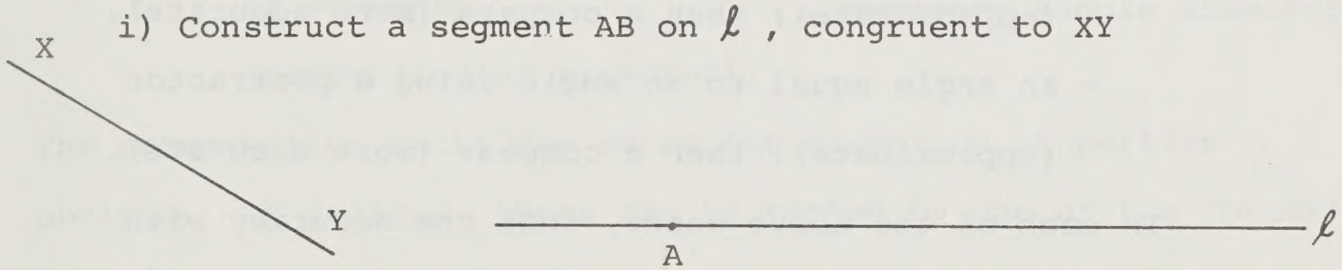
Draw the required line through P and the intersection of arcs 2 and 4.

The lines should be tested for parallelism.

Constructing Congruent Segments and Angles

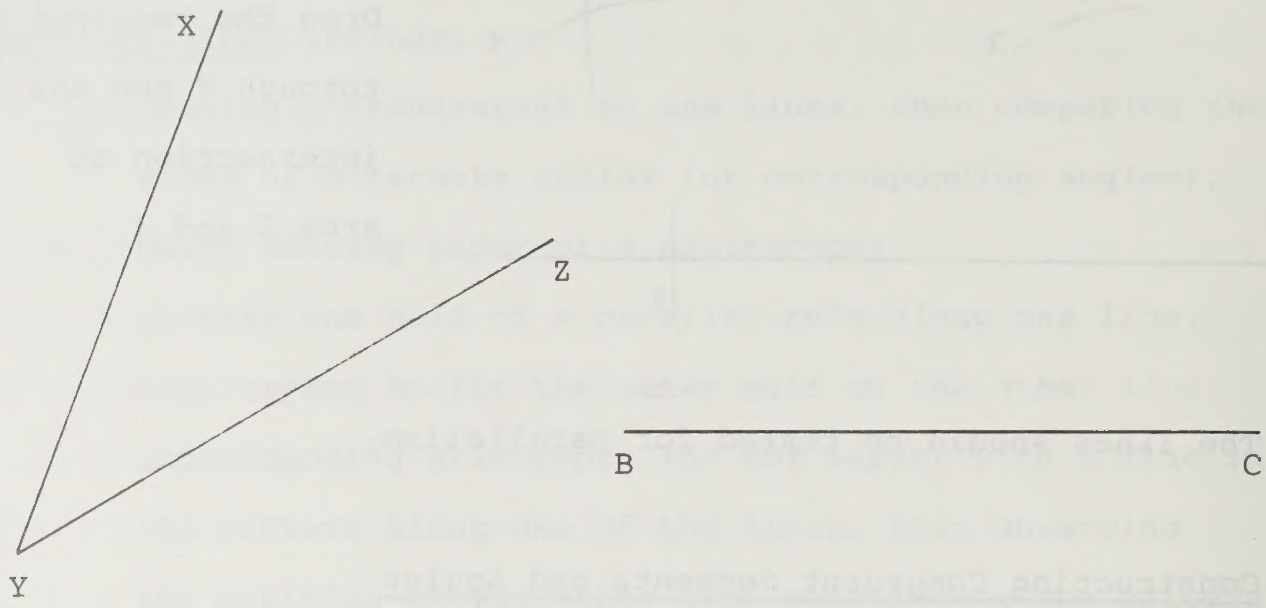
Techniques for using tracing paper to compare the lengths of segments and the sizes of angles are discussed on pages 7 and 8 of the notes for 7G 1b. A slight variation on these techniques provides ways of constructing a segment congruent to a given segment, and an angle congruent to a given angle. Examples are given below.

i) Construct a segment AB on ℓ, congruent to XY



- On tracing paper, mark the positions of points X and Y.
- Turn the paper over.
- Fit one point on A, the other point on ℓ. Mark the position of this point. This is B. (When the tracing paper is turned over, the pencil marks for X and Y act as carbon.)

ii) Construct an angle ABC equal to angle XYZ



- On tracing paper, mark the positions of X,Y, and Z.
- Turn the paper over.
- Fit the tracing of Y on B, of X on BC. Mark the position of the tracing of Y. This is point A.
- Draw AB to make ∠ABC.

Students should also practise constructing:

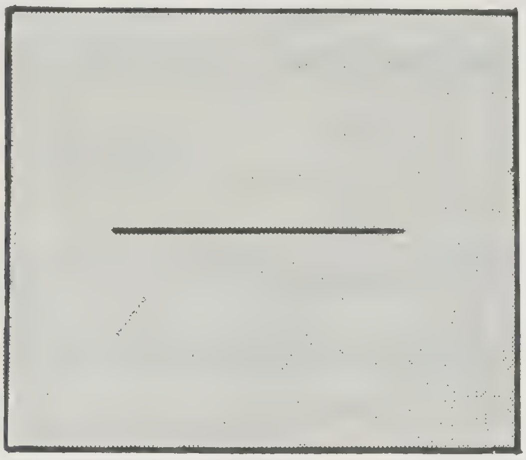
- a segment congruent to a segment using a ruler (approximate); then a compass (more accurate),
- an angle equal to an angle using a protractor (approximate), then a compass (more accurate).

In each of the above cases, test the accuracy with tracing paper.

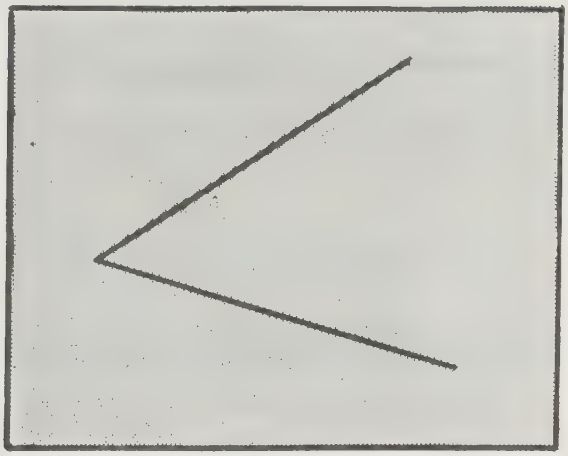
Perpendicular Bisector, Angle Bisector

The perpendicular bisector of a segment, and the bisector of an angle are lines of symmetry of these figures. This idea can be introduced by having each student construct the line of symmetry of a number of line segments and a number of angles. This can be done with tracing paper or with a transparent

mirror. The tracing paper methods are illustrated below.



- Trace the figure.
- Fold one end point onto the other and crease.
- Unfold & fit the tracing onto the segment.
- Pierce the crease twice (using compass point).
- Draw the required line of symmetry (perpendicular bisector).



- Trace the figure
- Fold one arm onto the other and crease
- Unfold & fit the tracing onto the angle.
- Pierce the crease at least once.
- Draw the required line of symmetry (angle bisector)

The construction technique makes the bisection properties obvious. If desired, these can be tested by one of the (other) methods mentioned earlier.

The traditional techniques using ruler and compass can now be developed and the results tested with tracing paper or a transparent mirror. The techniques for constructing a perpendicular and an angle bisector can now be combined to construct angles of 45° (135°), $67\frac{1}{2}^{\circ}$ ($112\frac{1}{2}^{\circ}$), and $22\frac{1}{2}^{\circ}$ ($157\frac{1}{2}^{\circ}$).

Students should recognize that it is not necessary to draw segment AB in order to construct its perpendicular bisector, regardless of the construction method used. This is significant in some of the applications that follow.

The construction techniques should be practised by building of triangles and quadrilaterals to specified criteria. (This is an introduction to topic c) below.)

For example, construct

- a right-angled triangle with legs of 6 cm and 9 cm;
- a right-angled isosceles triangle with legs of 10 cm;
- an equilateral triangle with sides of 7 cm;
- a scalene triangle with sides of 7, 9, and 12 cm;
- a rectangle with sides of 15 cm and 8 cm;
- a parallelogram with sides of 15 cm and 8 cm, and an angle of 45°;
- and so on.

As mentioned earlier, the students in any one class will likely have had a variety of experiences with the concepts and construction techniques of this section before entering Grade 8. The notes for 7G 2a) and for the present topic have suggested many ways of introducing, reinforcing, and applying these ideas. Every student should know some of these ways, some students (because of earlier experiences) should be challenged to study many of them. The teacher should observe the class carefully in order to suggest appropriate activities to appeal to different groups within the class.

Reference to the use of geometric constructions in different occupations and professions, and to simple applications drawn from the real world will add interest and relevance to this important topic.

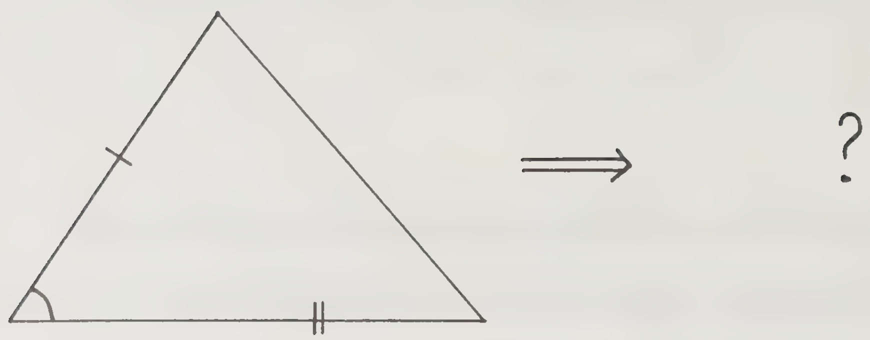
b) Constructing a triangle congruent to a given triangle; sufficiency conditions; constructing special cases of triangles and quadrilaterals

Constructing Triangles

Students, at this stage, are familiar with congruent figures. For example, they should know that if two triangles are congruent, a tracing of one will fit exactly on the other, and so the corresponding sides are congruent and corresponding angles are congruent. The present topic brings a different view to congruence, by introducing the idea of sufficiency conditions for congruence through the media of accurate constructions.

Given a triangle in which certain conditions are known, the student is challenged to construct a triangle congruent to the given triangle. For example, given $\triangle ABC$ below, the student could be asked:

- to try to construct $\triangle DEF$ in which $DE = AB$, $EF = BC$, and $\angle DEF = \angle ABC$
- if this is possible, to test these triangles for congruency (using tracing paper, or by measuring corresponding sides and angles).



This problem is trivial when using tracing paper or a transparent mirror, since the technique that would be used guarantees congruence. The problem should therefore be attempted with compass and ruler (and possibly with straight edge and protractor, as a lead in technique for some students). The results of this investigation by each student should be compared by group or class discussion.

Now the cases of

i) side, angle, angle;

ii) side, side, side;

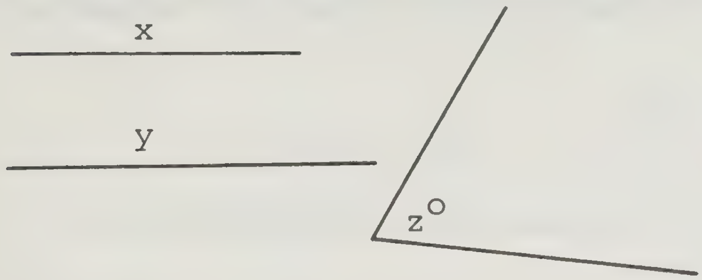
iii) side, right angle, hypotenuse;

iv) side, side, angle;

v) angle, angle, angle

may be investigated. Cases i), ii), and iii) give congruent triangles, iv) yields two possible triangles, and v) many triangles (infinitely many, similar triangles).

A variation on these examples can now be investigated. This time provide the students with two segments and an angle

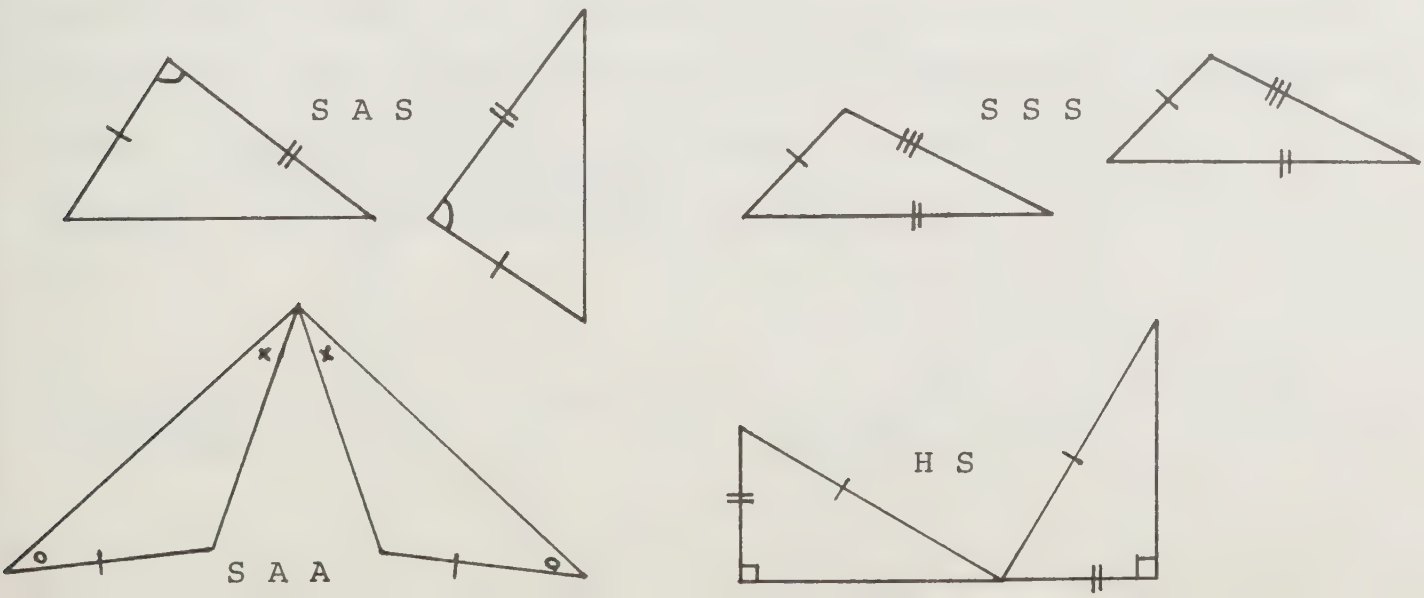


and ask each student to try to construct a triangle with sides of lengths x and y and the contained angle of size z°. The use of tracing paper, or a transparent mirror, is not trivial this time; nor is the use of a compass and straight edge.

This variation could now be examined for cases i) to v) above.

<u>Sufficiency Conditions</u>

The above investigations should lead to a discussion of 'sufficiency conditions' for a unique triangle; that is, the information that must be known in order that one triangle AND ONLY ONE triangle, can be constructed to satisfy the given information. In turn, this discussion should lead to the familiar conditions for congruent triangles; that is SAS, SSS, SAA, and HS (hypotenuse H, side S, -- H implies a right angle).

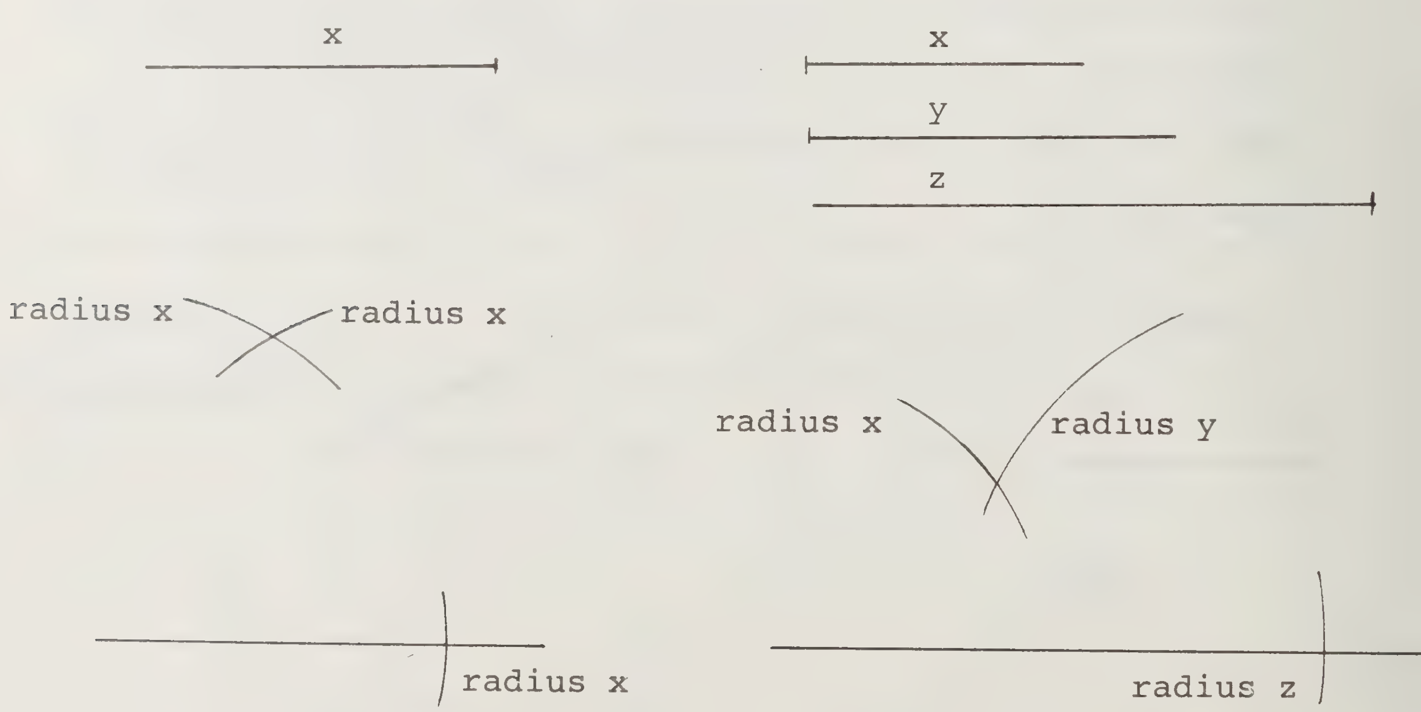


Given two triangles that satisfy one of these conditions, the student should be able to identify the other corresponding parts of the two triangles.

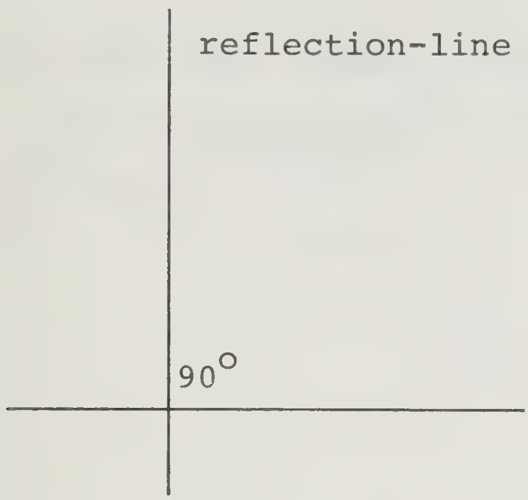
<u>Constructing Special Cases of Triangles and Quadrilaterals</u>

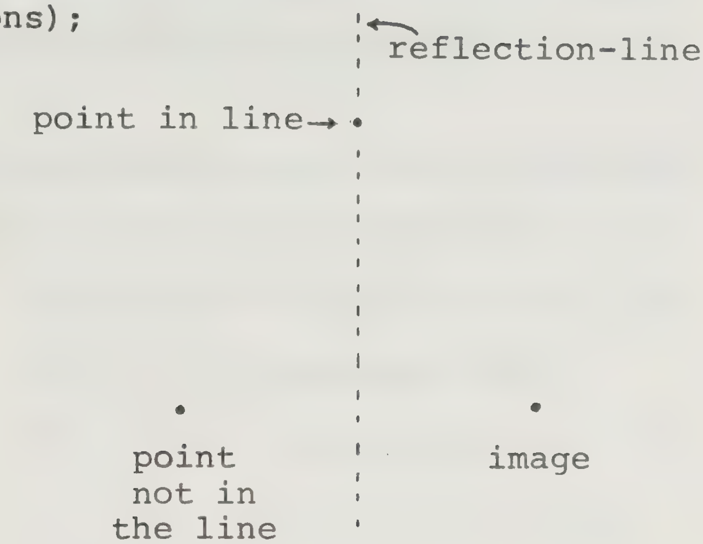
This topic combines the construction techniques of 8G 1a with the properties of 7G 1d (in which triangles and quadrilaterals are classified by their side-angle properties and by their symmetry properties).

Students should be asked to construct isosceles triangles, right-angled isosceles triangles, right-angled triangles, equilateral triangles, and scalene triangles that satisfy certain conditions. (These are sufficiency conditions for a unique triangle.) These figures can be constructed using a variety of instruments. In some cases, a compass-straight edge construction provides the simplest approach (equilateral triangle, triangle given the lengths of the sides);

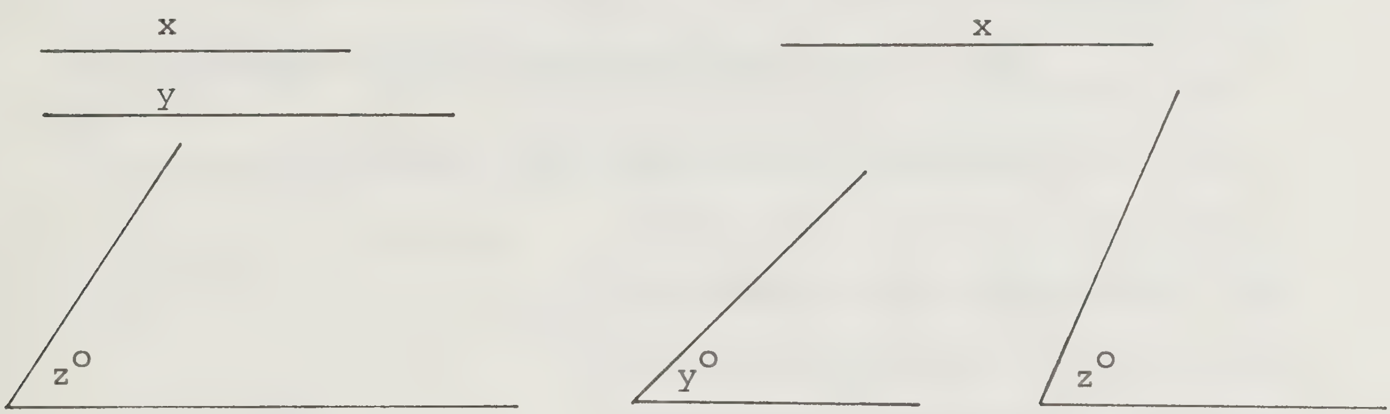


in other cases, a transparent mirror (right-angled triangles, isosceles triangles, combinations);





in other cases, tracing paper (SAS, SAA)



and in other cases, combinations of two or more of these techniques.

When constructing special quadrilaterals, the same considerations apply. The simplest technique sometimes is with straight edge and compass, sometimes a transparent mirror, sometimes tracing paper, and often a combination of these. The case for SSSS might be considered, the quadrilateral is not rigid (not unique); this is suggested for discussion in 8G 3c.

c) Constructing a circle; locating the centre of a circle and of a circular arc; concentric circles

This topic introduces simple constructions related to a circle. The student should be able to construct a circle (or circular arc), when given its centre and

i) its radius;

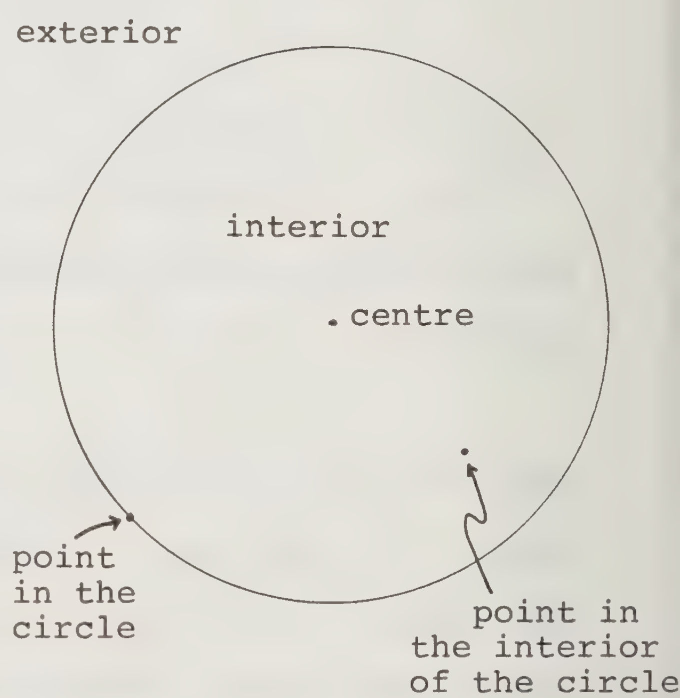
ii) its diameter;

iii) a point in it.

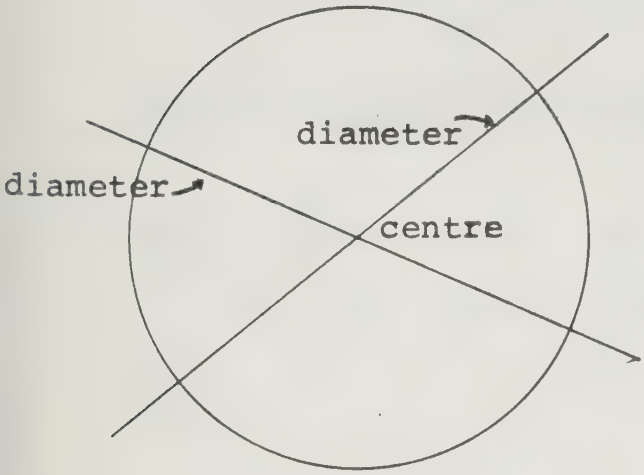
The Fundamental Property of a Circle is as follows.

The distance is constant between the centre of a circle and any point in it.

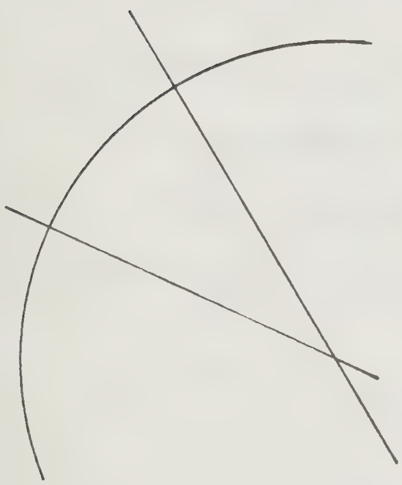
Any one who has used a compass to draw a circle will find that this property is obvious. Note that a circle is defined as the set of all points equidistant from a fixed point (the centre of the circle). A 'point in the circle' is a point of this set



A 'point in the interior of a circle' is a point in the region that is enclosed by the circle.

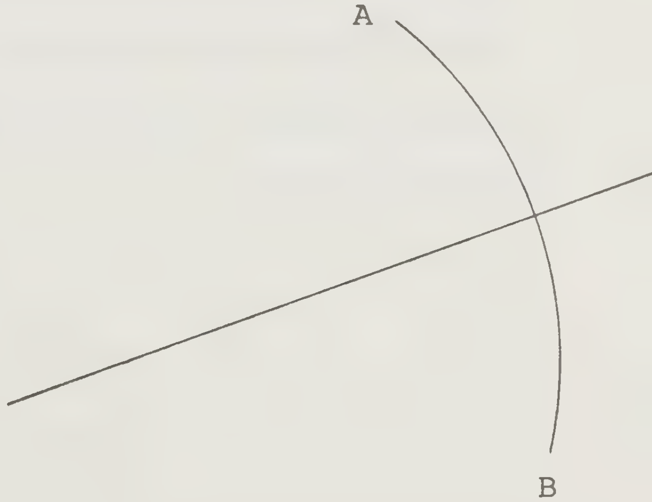


The centre, as the intersection of two or more diameters, can be located by paper folding or with a transparent mirror. This leads to the notion of the infinite line-symmetry of a circle.

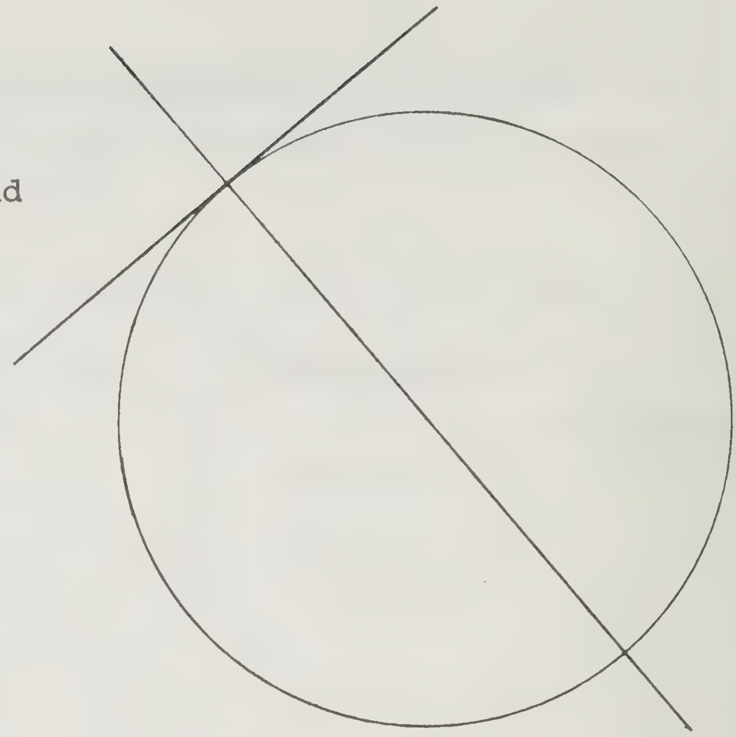


The centre of a circular arc can be located by folding (or reflecting) the arc onto part of itself, at least twice. The fold-lines (or reflection-lines) intersect at the centre, and are perpendicular to the arc.

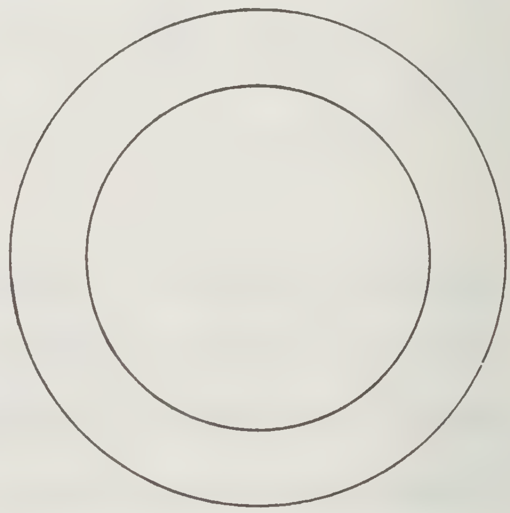
Fold (or reflect) the end of a circular arc onto the opposite end. The fold-line (reflection-line) is the perpendicular bisector of the circular arc.



fold or reflect
B onto A



This topic can be extended by constructing a tangent at the intersection of the diameter and the circle (perpendicular to the diameter). Then ideas of motion along curved paths can be introduced; the direction of the motion at any instant is along the tangent to the curve at that point. This has real world applications to such situations as: the direction of the path of a ball as it is released from a person's hand, the direction that a space capsule is travelling at any instant, and the direction a human body moves when propelled from a vehicle in a crash. When David slew Goliath, the stone was propelled tangentially from the circular path of the sling.



Concentric circles and concentric arcs pass the same test for parallelism as do straight lines (using tracing paper or a transparent mirror).

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GRADE 8 GEOMETRY

SECTION 2: CONGRUENCE TRANSFORMATIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Page 12; slide, turn, flip, symmetry
	Ed PJ Div:	Pages 74-76
	Gr 7:	N 8; G 1; G 2ab; G 3; G 4; G 5; G 6e
PRESENT	Gr 8:	G lac; G 2; G 3ad; G 4; G 5d
FUTURE	Gr 9 Gen:	A 4a; G lac; G 2bc; G 3
	Gr 9 Adv:	A 4a; G 1; G 2b; G 3; G 4ab
	Gr 10 Gen:	G 1
	Gr 10 Adv:	N 4bcf; G 2; G 3; G 4; G 6cd

a) Constructing translation-, rotation-, and reflection-images from the fundamental properties

This topic extends ideas that have been introduced in the K - 7 program. The notes for 7G 4 present various ways of obtaining slide-, turn-, and flip-images of a figure (pages 2 to 13) and then establish the mathematical meaning of translation, rotation, and reflection and their fundamental properties (pages 14 to 18). The present topic uses these properties as the basis for constructing images accurately in a variety of ways. The images may be constructed with a compass and ruler, tracing paper, transparent mirror, or ruler (same scale on both edges) and protractor.

1. Using a Compass and Ruler

i) Translation

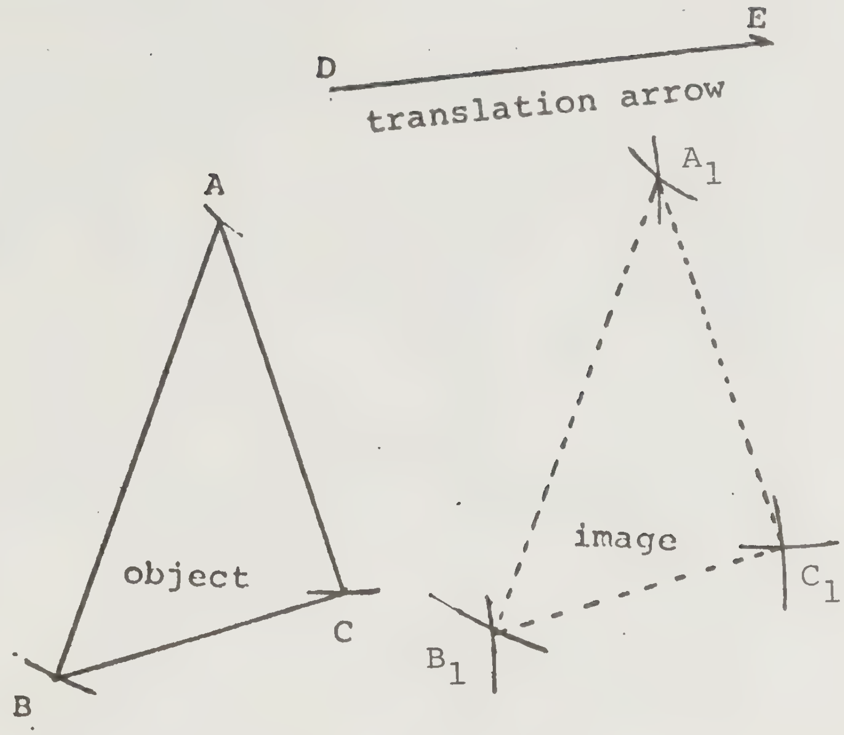
The Fundamental Property of a Translation:

Under a translation the arrow from any point P to its image-point P_1 has the same length and same direction as the arrow from any other point Q to its image point Q_1

Construct the image of $\triangle ABC$ under the translation defined by arrow $\overrightarrow{DE}$. (This method uses the compass and ruler technique for constructing a parallelogram — opposite sides are equal and parallel.)

Method

- Locate A_1 as the fourth vertex of parallelogram $ADEA_1$. (Arrow $\overrightarrow{AA_1}$ has the same length and direction as arrow $\overrightarrow{DE}$.)
- Locate B_1 and C_1 in the same way.
- Draw $\triangle A_1B_1C_1$, the required translation image of $\triangle ABC$.



ii) Rotation

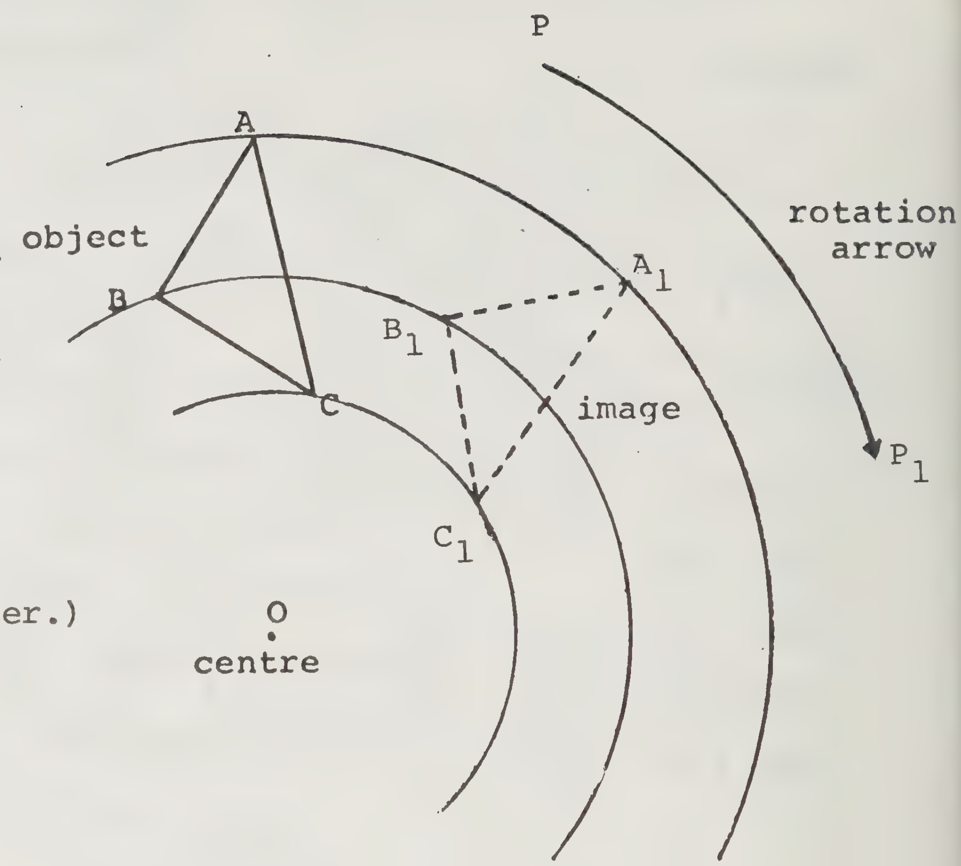
The Fundamental Property of a Rotation:

Under a rotation with centre O i) each point P in the object is the same distance from O as its image P_1 ii) angle POP_1 equals the angle of rotation for every position of P.

Construct the image of $\triangle ABC$ under the rotation with centre O and angle determined by the given rotation arrow (this method uses concentric arcs and congruent angles).

Method

- Draw concentric arcs with centre O through A, B, and C.
- Locate A_1 on the outer arc so that angle AOA_1 equals the rotation angle POP_1. (This can be done with a protractor or tracing paper.)
- Locate B_1 and C_1 in the same way.
- Draw $\triangle A_1B_1C_1$, the required rotation image of $\triangle ABC$.



iii) Reflection

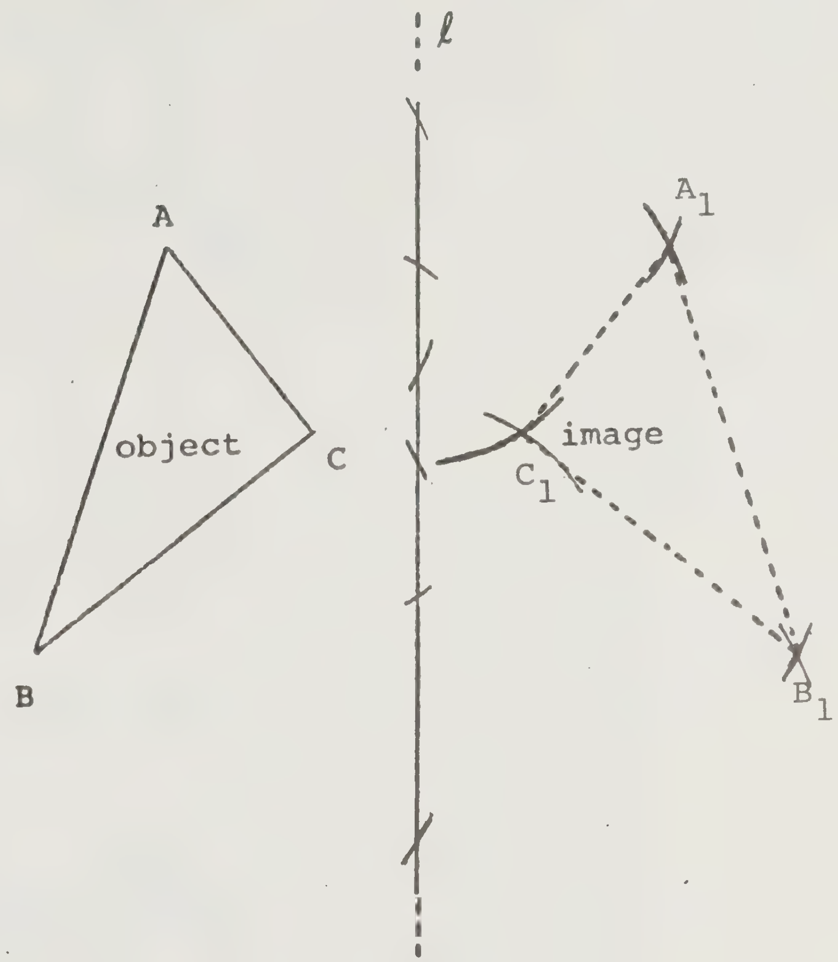
The Fundamental Property of a Reflection:

> Under a reflection in line m,
> line m is the perpendicular bisector of the segment joining each point P to its image P_1 (m is the perpendicular bisector of PP_1).

Construct the image of $\triangle ABC$ under reflection in line ℓ. (This method uses the property that the diagonals of a rhombus perpendicular bisect each other.)

<u>Method</u>

- Locate A_1 so that ℓ is the perpendicular bisector of AA_1 (A_1 is the vertex of a rhombus opposite A.)
- Locate B_1 and C_1 in the same way.
- Draw $\triangle A_1B_1C_1$, the reflection image of $\triangle ABC$ in line ℓ.



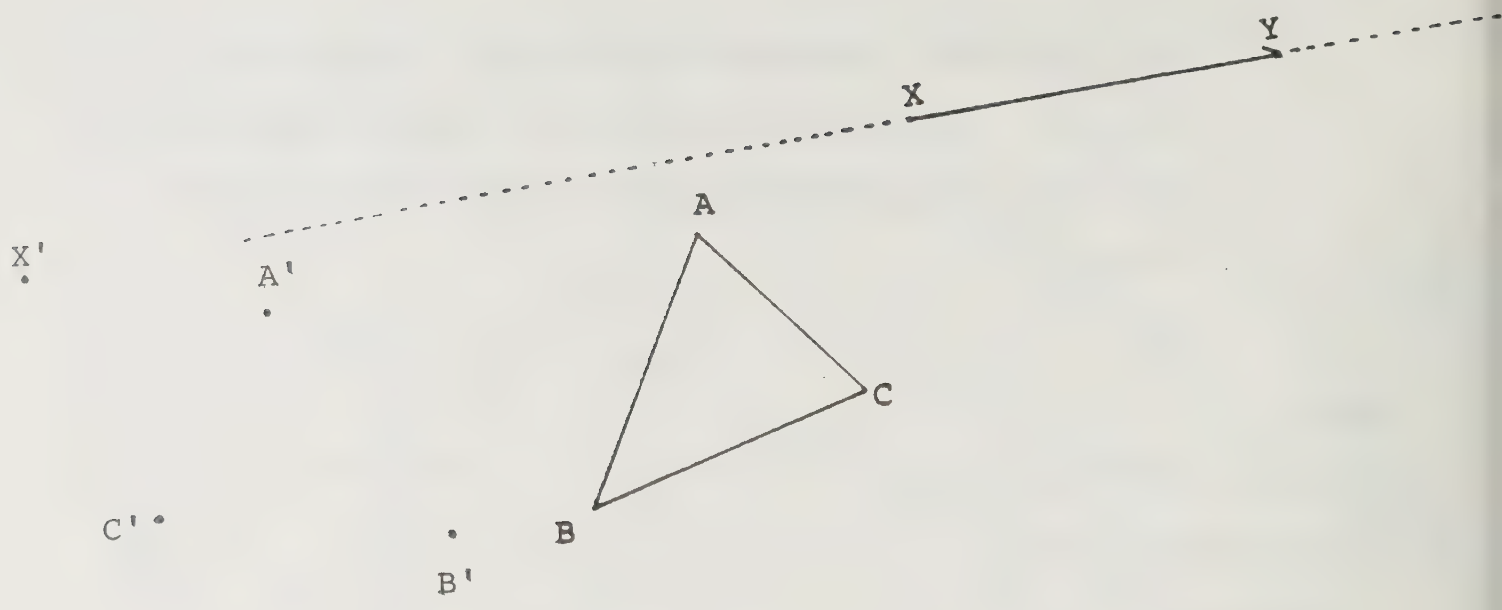
2. <u>Using Tracing Paper</u>

Techniques for using tracing paper for accurate construction of translation-, rotation-, and reflection-images are given on pages 11-18 of the notes for 7G 4.

3. <u>Using a transparent mirror</u>

i) <u>Translation</u>

Construct the image of $\triangle ABC$ under the translation defined by arrow $\overrightarrow{XY}$.

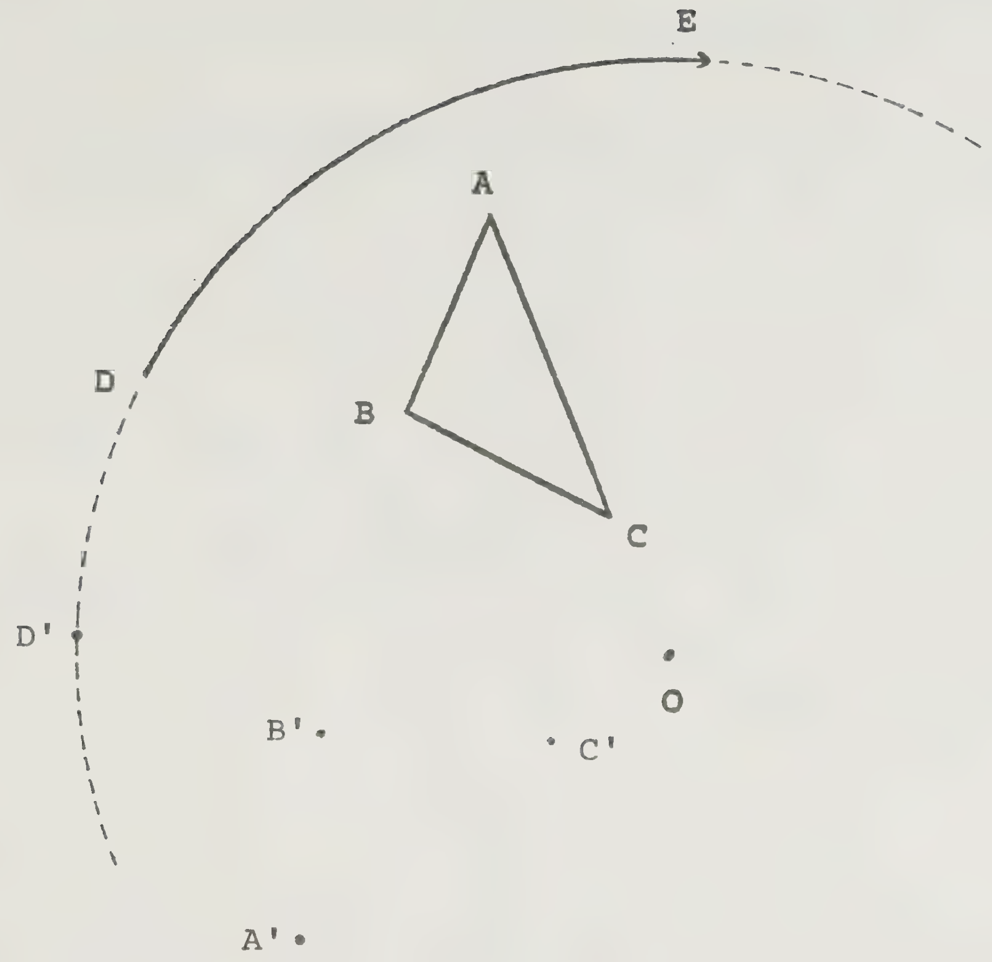


Method

- Extend the arrow by a 'dashed' line.
- With the mirror perpendicular to XY and to the left of B, locate the images of A, B, C, and X. (These are labelled A', B', C', and X' in the diagram; A' is called <u>A prime</u>.)
- Looking in the mirror from the other side, reflect X' onto X. (The image of X' is called X"; observe A", B", C" on A, B, and C; X" is called <u>X double prime</u>.)
- Move the mirror so that X" slides along the arrow to Y. (Observe A", B", C" sliding the same distance in the same direction.)
- Mark the positions of A", B", and C". Label these points A_1, B_1, and C_1.
- Draw $\triangle A_1B_1C_1$, the required translation-image of $\triangle ABC$.

ii) <u>Rotation</u>

Construct the image of $\triangle ABC$ under the rotation with centre O and turn arrow DE.

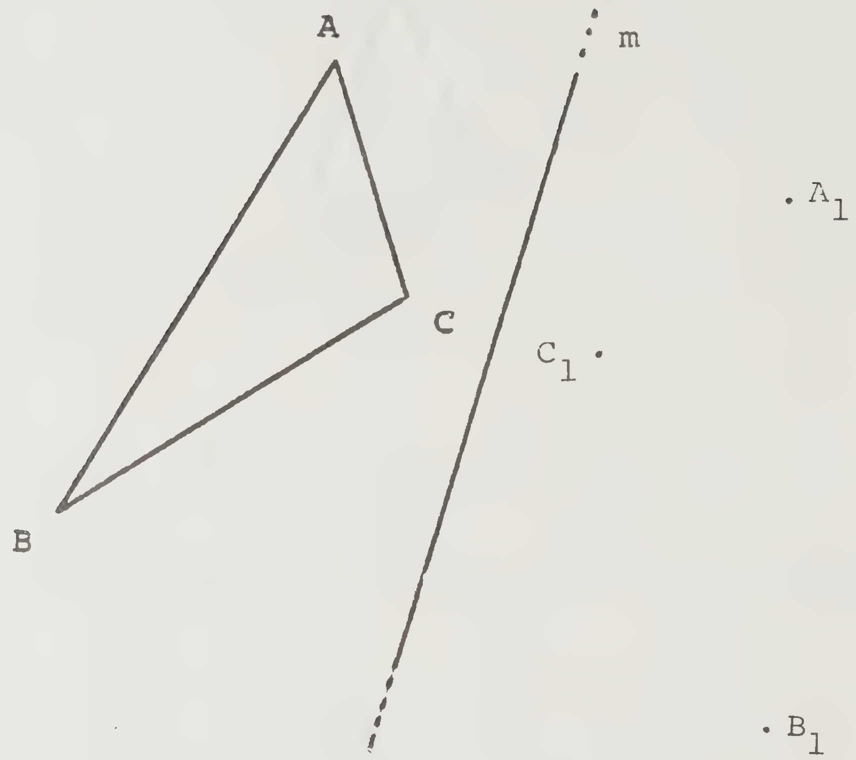


Method

- Extend the turn arrow by a dashed arc.
- With the mirror perpendicular to the arc and counter clockwise from D, locate the images of A, B, C, and D. (These are labelled A', B', C', and D' in the diagram.)
- Looking in the mirror from the other side, reflect D' onto D. (The image of D' is D"; observe A", B", and C" on A, B, and C.)
- Turn the mirror so that D" moves along the turn arrow to E. (Observe A", B", C" turning about O.)
- Mark the positions of A", B", C". Label these points A_1, B_1, and C_1.
- Draw $\triangle A_1B_1C_1$, the required rotation-image of $\triangle$ ABC.

iii) Reflection

Construct the image of $\triangle ABC$ under reflection in line m.

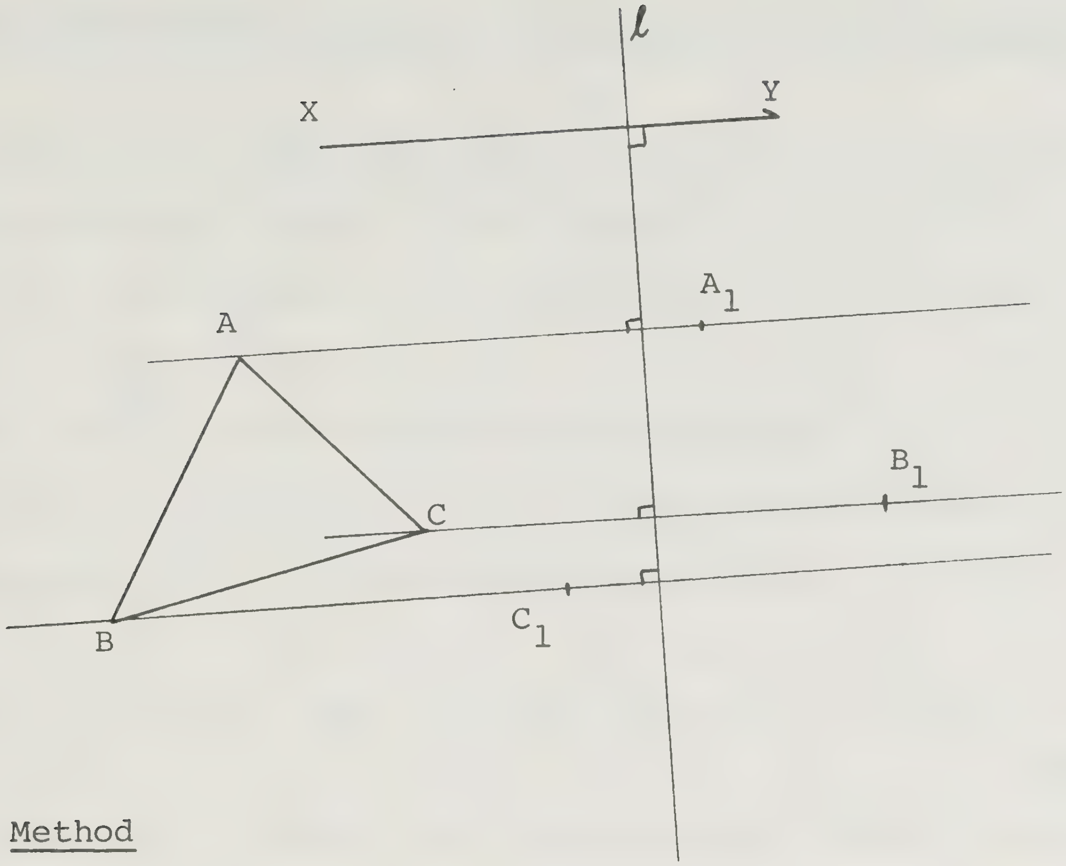


Method

- Stand the mirror with its edge along line m.
- Locate the images of A, B, and C. Label them A_1, B_1, and C_1.
- Draw $\triangle A_1B_1C_1$, the required reflection-image of $\triangle ABC$.

4. Using a Ruler and Protractor

Translation (using a ruler with the same scale on both edges)



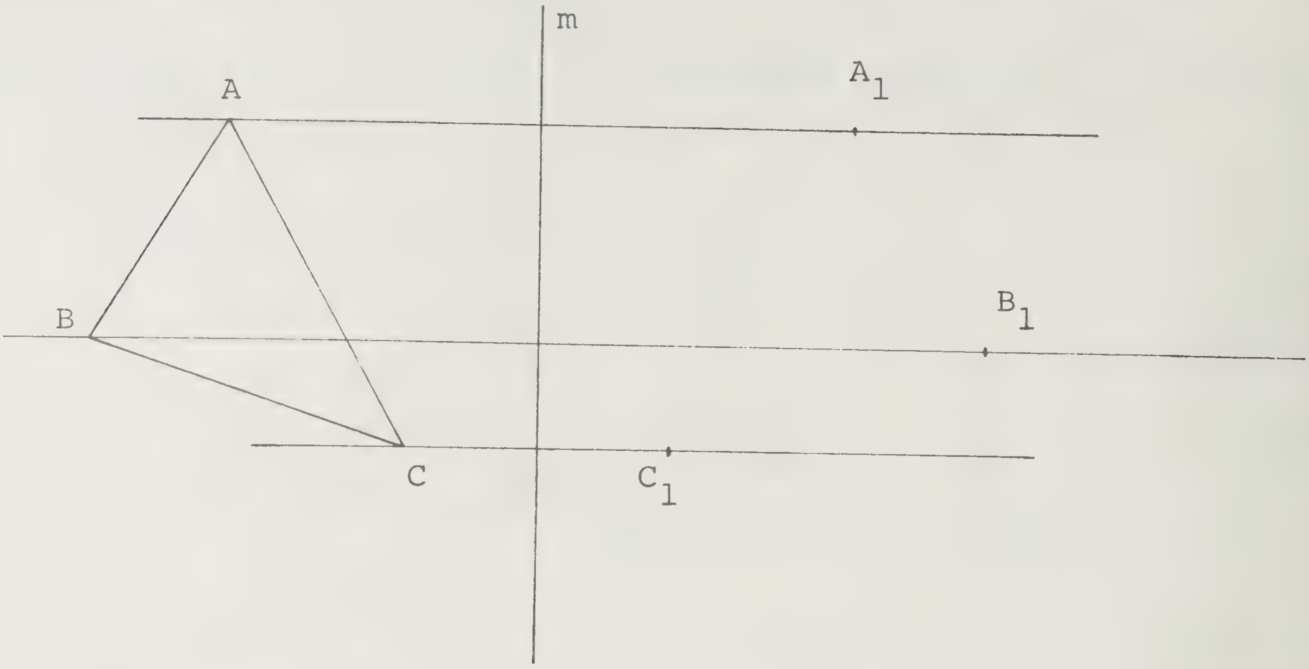
Method

- Use the ruler to draw a line ℓ perpendicular to the arrow.
- Use the ruler to draw lines through A, B, and C perpendicular to ℓ .
- By measurement, locate A_1, B_1, and C_1 on these lines so that $AA_1 = BB_1 = CC_1 = XY$. (This method usually gives less accuracy than the previous methods.)

Rotation (using a ruler and protractor)

This method is tedious and leaves many extra lines on the figure.

Reflection (using a ruler with the same scale on both edges)



Method

- Use the double scale to draw lines through A, B, and C perpendicular to line m.
- By measurement, locate A_1, B_1, C_1 so that m bisects AA_1, BB_1, and CC_1.

It is not expected that students should learn all the above construction techniques. The techniques with compass and ruler utilize the fundamental properties of the transformations and provide some useful practice with the techniques for drawing a parallelogram (translation) and a perpendicular to a line (reflection). The techniques with tracing paper are fast and give accurate results. The transparent mirror is fast and accurate for a reflection image, and has good visual impact for translation and rotation images. Regardless of method or methods used, a major purpose of this activity is to generate accurate diagrams so that the students may investigate other properties of each of these transformations. Each student should also settle on the technique or techniques that he/she will use to generate images in the future.

Other Properties

Properties of these transformations can be investigated by construction of images of line segments, angles, different polygons, and circles, and observation of properties that do not change (are invariant) and those that do. The student will find for each of these transformations that the image is congruent to the object. This is why they are called congruence transformations. Of course, this result will obviously occur if the constructions are made using tracing paper. The result is not so obvious when ruler and compass, or mirror, techniques are used.

Special properties occur for each of the transformations, such as the following:

> For translation, a segment and its image are parallel.
> For rotation, a segment and its image form an angle equal to the angle of rotation (they may have to be extended).
> For reflection, a segment and its image meet in the reflection-line (they may have to be extended), and form an angle that is bisected by the reflection-line.

There are many other interesting properties of these transformations. These will be investigated more systematically in 9 Adv G 3a.

Students might enjoy finding how reflection is used in planning strategies in billiards and miniature golf.

b) <u>Reflection-, translation-, and rotation-images in the real world</u>

There are many situations related to visual reflection-images in real life. Images occur in flat and curved mirrors; some of these are congruent to the objects (in flat mirrors), others are distorted from the object (curved and irregular mirrors). Students could place various geometric figures in front of mirrors of different kinds and observe the properties of the images (those that change, those that do not).

Write your name and draw other figures on tracing paper. When viewed from the back you will see reflection images.

When you see PULL on a glass door, be sure to push.

If the image car in the rear-view mirror has its right blinker flashing, then the car is going to turn left.

Why is AMBULANCE printed on the front of an ambulance?

When cutting carpet (or vinyl tile) from the back, be sure to cut on the reflection image of the floor plan.

Examine and explain the nature of the printing on a rubber stamp.

The list goes on and on; the students can find other instances.

The world is filled with objects that slide, turn, and/or flip. Congruence transformations help us to make diagrams to represent the before and after positions of these objects and to study the characteristics of the motion. In doing so, we are building 2-D models that represent real situations of our world.

c) Tests to show whether two congruent figures are related by reflection, translation, or rotation; constructing the reflection-line, translation arrow, or rotation centre and angle

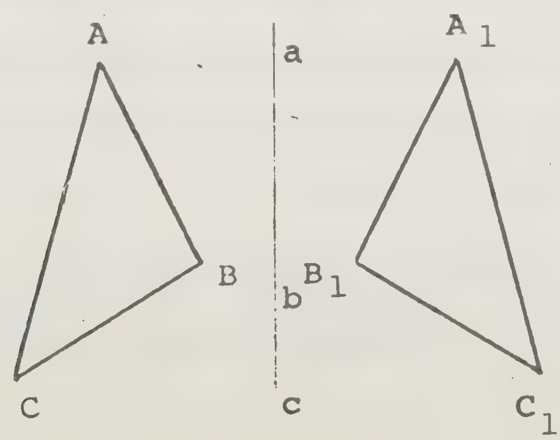
If two (plane) figures are congruent, there are only four transformations by which they can be related -- the three mentioned in the title above and glide-reflection. This is discussed in the notes for 10 Adv G 3b. At this time, students are not expected to be familiar with glide-reflection. Thus care must be taken when investigating the present topic to supply students with congruent figures that are related by only one of the above transformations. This is illustrated below for three sets of congruent triangles.

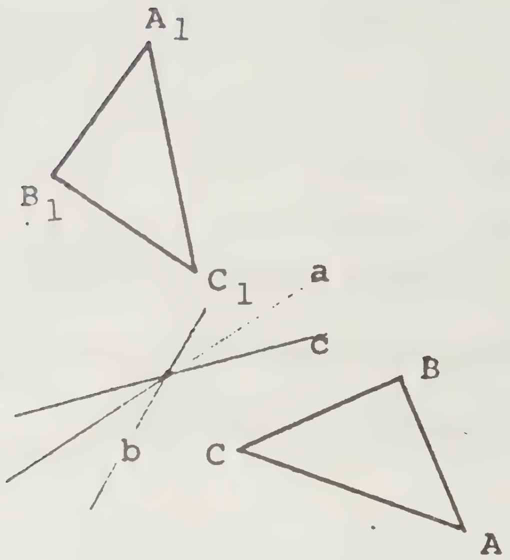
For each pair of triangles below:

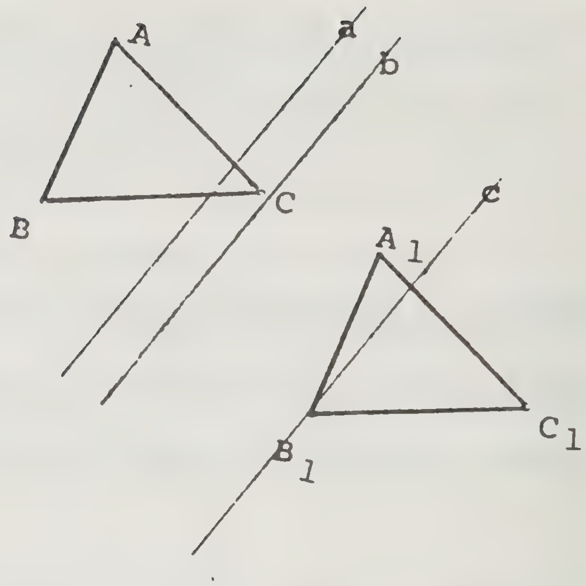
i) Draw the perpendicular bisector of a pair of corresponding vertices, A and A_1. Label it a.

ii) Repeat for B and B_1, then C and C_1. Label them b, and c.

The results are illustrated in the diagrams below.







In the first case, the three lines coincide; the triangles are related by reflection. The common line is the reflection-line. A transparent mirror will show this at a glance; alternately, test this with tracing paper.

In the second case, the three lines are concurrent (they meet in one point). The triangles are related by rotation; the centre of rotation is the point of concurrency. This can be demonstrated using tracing paper. Trace one triangle, pin the tracing paper at the point of concurrency, then turn the tracing until it fits on the other triangle. The tail and head of a rotation arrow can now be found using the tracing paper. The arrow can be drawn with a compass. This method can now be related to pictures of real world objects (car doors, brake pedals, etc.) in the <u>before</u> and <u>after</u> positions, to locate the centre of rotation. on.

In the third case above, the three lines are parallel. (This can be tested with a transparent mirror or tracing paper.)

The translation arrow can be shown as $\overrightarrow{AA_1}$, or any arrow with same length and direction as $\overrightarrow{AA_1}$.

d) Reflection images in curved mirrors and flat mirrors: 'reflection in a line' as a model of reflection in a flat mirror

Creating and playing with models of real things is instinctive for most people — both young and old. Children play with toys or dolls, and imagine they have the real thing. Children make drawings of the things they see. Many adults doodle, and sketch the things they see, often charmingly distorted. All these are representations of some aspects of reality. In the Intermediate Division Mathematics Program, a fundamental aim is to help the students represent aspects of the real world through drawings and formation of simple formulas and algebraic expressions.

'Reflection in a line' is one example of a model of reflection in a mirror of the real world. The model and the real world situation have many characteristics in common. For example, a figure and its image are congruent in both cases. In the real world, a reflection-image is turned 'inside-out' (think of a glove and its image); in reflection in a line, a figure is 'flipped over' to form the image. Reflection in a curved mirror, such as a chrome tea kettle, produces a number of effects such as distorting the image, making the image seem smaller and closer to the reflecting surface than the object, and bending line segments. Obviously the images we draw under reflection in a straight line would be a poor representation of the images we see in the kettle, a poor mathematical model.

Ministry of Education Ontario

GRADE 8 GEOMETRY

SECTION 3: PROPERTIES OF PLANE FIGURES

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7, 12
	Ed PJ Div:	Pages 75, 76
	Gr 7:	N 8a-d,g; A 1b; G 1; G 2; G 3; G 4; G 5; G 6
PRESENT	Gr 8:	N 6abc; G 1; G 2; G 3; G 4; G 6
FUTURE	Gr 9 Gen:	N 3c; N 6bd; G 1; G 2; G 3; G 4
	Gr 9 Adv:	N 5cef; N 6a; G 1; G 2abef; G 3; G 4
	Gr 10 Gen:	N 2ab; N 4c; N 7; G 1; G 2c
	Gr 10 Adv:	N 2; G 1a-e; G 3; G 4; G 5e; G 6c

a) <u>Properties of triangles, parallel lines, and quadrilaterals; the Pythagorean Theorem</u>

Students entering Grade 8 are usually familiar with many properties of triangles, parallel lines, and quadrilaterals as a result of previous activities with concrete materials in which they investigated ideas of symmetry, slides, turns, flips, classification, constructions, measurement, and tiling. Many ideas in the earlier notes for Grades 7 and 8 could provide interesting ways for 're-discovering' the properties. The purpose of this topic is to draw together the properties of specific figures.

<u>Triangles</u>

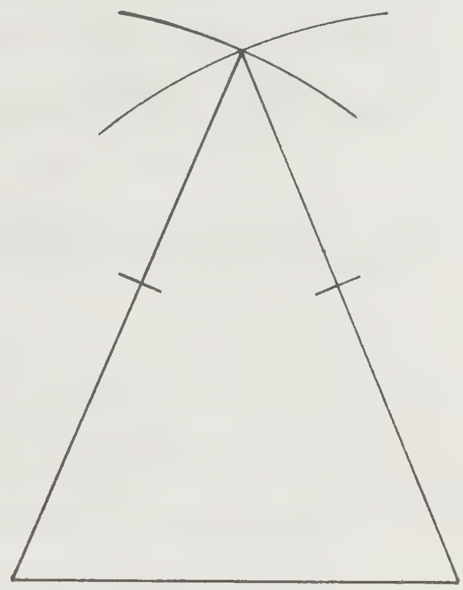
Properties of triangles can be investigated by measuring with a ruler and a protractor, by using tracing paper, by using a transparent mirror, or by examining tiling patterns made from equilateral, isosceles, and scalene triangular tiles (see the notes for 7G 3a,b,d,e). The notes that follow suggest a few methods for investigating the properties; they are not exhaustive.

Students should already be familiar with the classification of triangles by the nature of the largest angle, by the number of equal sides, and by the number of lines of symmetry. See the notes for 7G 1d pages 18 to 20.

As a general approach to this topic, the students could accurately construct special types of triangles and then determine their other properties -- by measuring, or by using tracing paper or a transparent mirror, or by a combination of these methods. This approach applies ideas that are introduced in 7G 1d, 7G 2, and 8G lab.

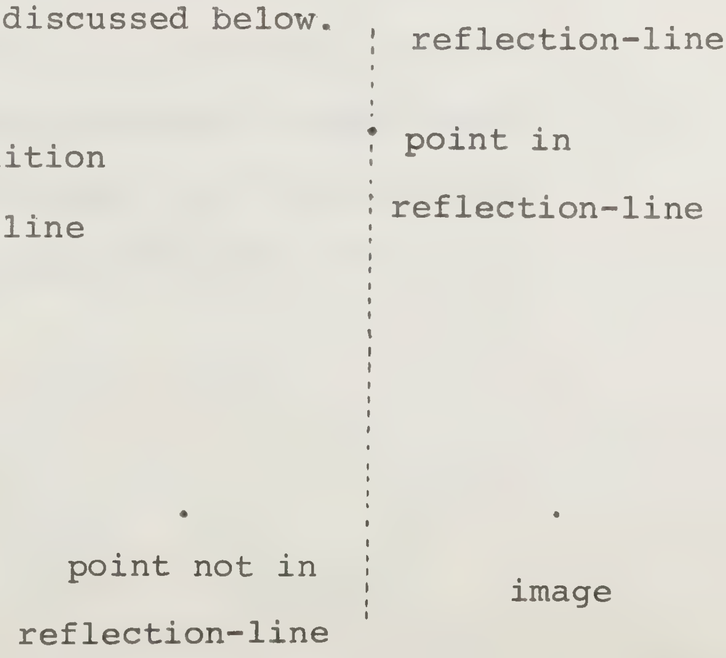
1. Isosceles Triangles

If the students use the definition 'an isosceles triangle has two equal sides', they will likely use ruler and compass, as illustrated, to construct their triangles. The sides are equal because the radii of the arcs are equal.



From this point, they can 'discover' that the triangle has two equal angles (called base angles). By using a tracing or a transparent mirror, they could find that the triangle has a line of symmetry and then draw this line. This would lead to other properties, as discussed below.

If the students use the definition 'an isosceles triangle has a line of symmetry', then they will likely use a transparent mirror or tracing paper to construct triangles.

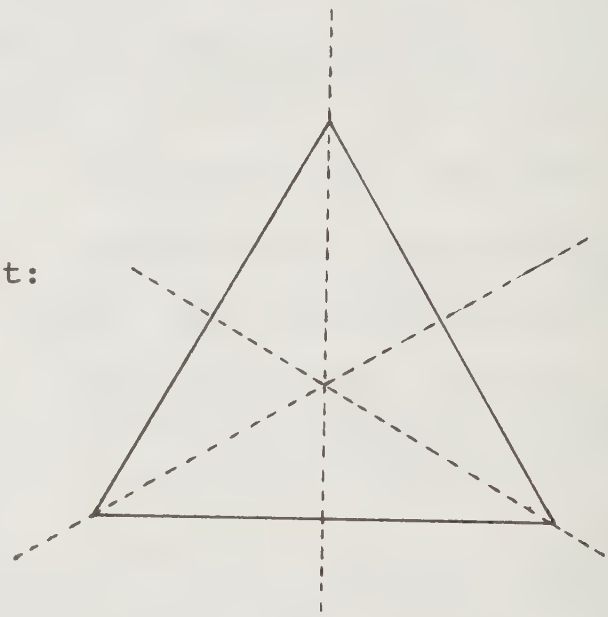


By drawing the line of symmetry (or looking in the mirror) they will locate the corresponding congruent parts and thus determine:

- two congruent sides;
- two congruent angles;
- the line of symmetry bisects the base perpendicularly;
- the line of symmetry bisects the third angle.

These properties can be labelled on the diagram. See the notes for 7G 3 pages 4, 5, 11, and 12.

2. Equilateral Triangles



An activity similar to the one above will reveal that:

- three sides are congruent;
- three angles are congruent (each 60°);
- there are three lines of symmetry (concurrent);
- each line of symmetry bisects a side perpendicularly;
- each line of symmetry bisects an angle;
- the lines of symmetry divide the equilateral triangle into six congruent triangles, with angles of 30°, 60°, and 90°;
- an equilateral triangle has rotational-symmetry (with centre the point of concurrency of the lines of symmetry; and of size 120°, 240°, and of course 360°).

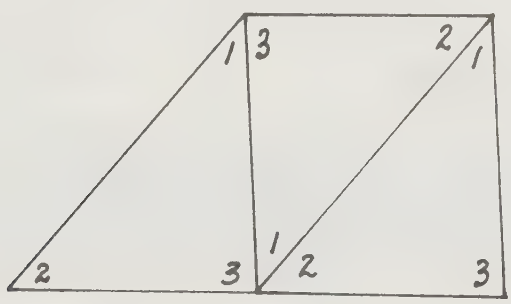
3. Scalene triangles

A scalene triangle does not have equal sides, equal angles, or lines of symmetry. The properties developed below for scalene triangles will hold for all triangles.

i) The sum of the angles in a triangle is 180°

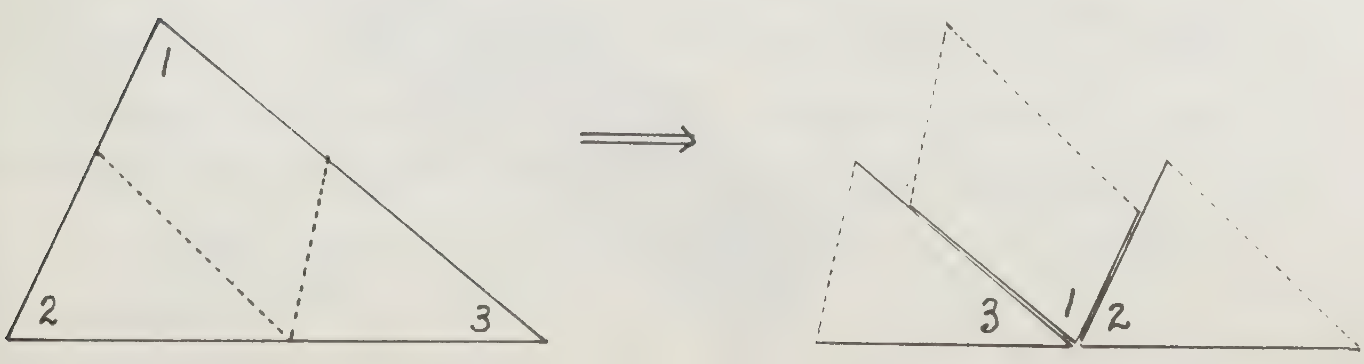
. Draw a triangle. Measure its three angles. Total the measures. Repeat for two or three triangles. Discuss with the class and compare results.

. Fit three congruent triangle tiles so that the three different vertices touch and the sides are edge to edge, as illustrated. See the notes for 7G 3, pages 9 and 20



. Examine tiling patterns made with triangles. See 7G 3, page 20.

. Draw a triangle on scrap paper. Cut it out with scissors. Cut off the corners (as illustrated). Fit the three corners, vertex to vertex.



. Draw any triangle ABC (figure 1).

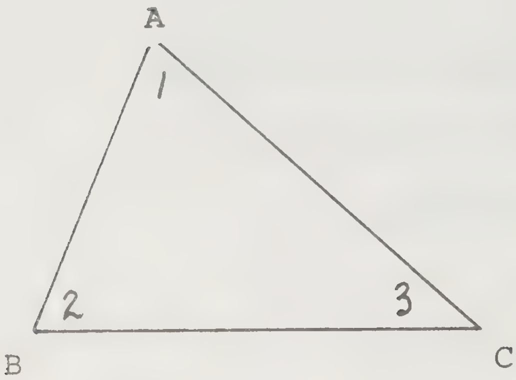


Figure 1

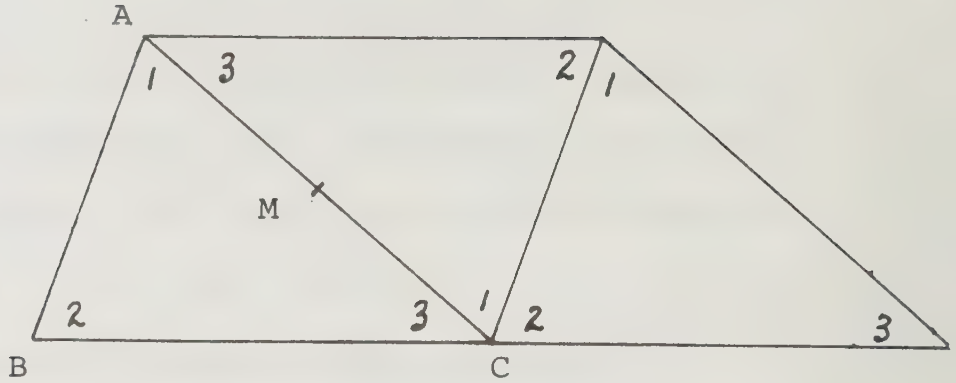


Figure 2

Draw the image of $\triangle$ABC under translation with arrow $\overrightarrow{BC}$.
Draw the half-turn image of $\triangle$BAC about the mid point of AC. (The image of BA under the half-turn will coincide with the image of BA under the translation $\overrightarrow{BC}$ (as already drawn).)

. Draw any triangle ABC. Draw a long arrow on tracing paper.

Place it as shown in figure 1.

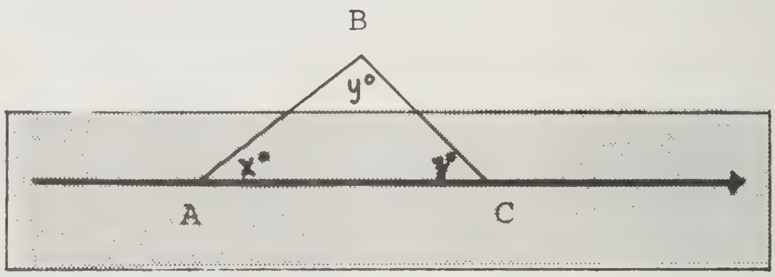


figure 1

Turn the arrow about A, through x^{o}. See figure 2.

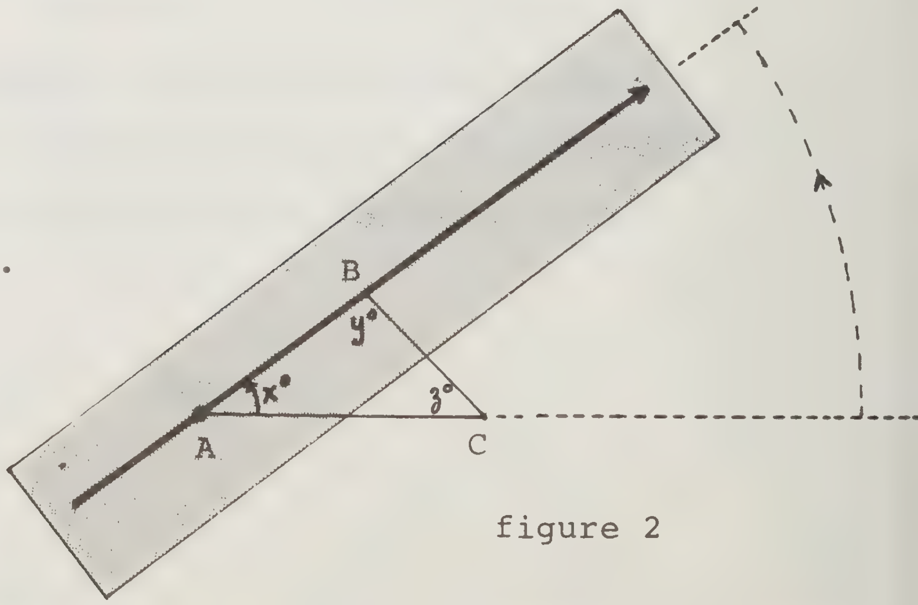


figure 2

Turn the arrow about B, through y°. See figure 3.

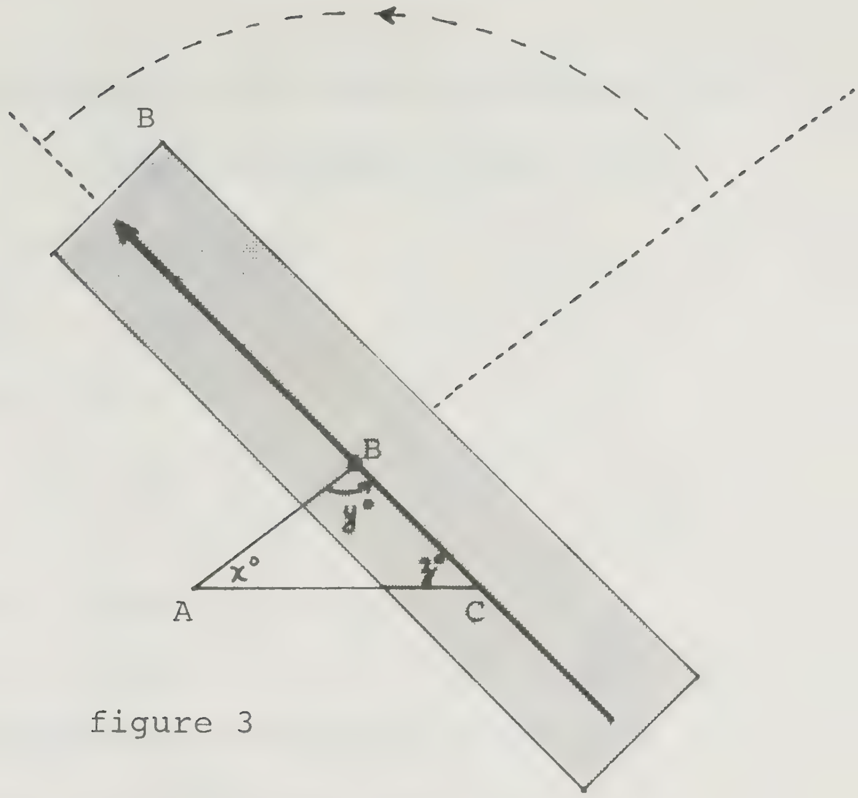


figure 3

Turn the arrow about C through z°. See figure 4.

The arrow now faces left; it has turned through 180°. It has also turned through $(x + y + z)^{\circ}$. Thus $x + y + z = 180$.

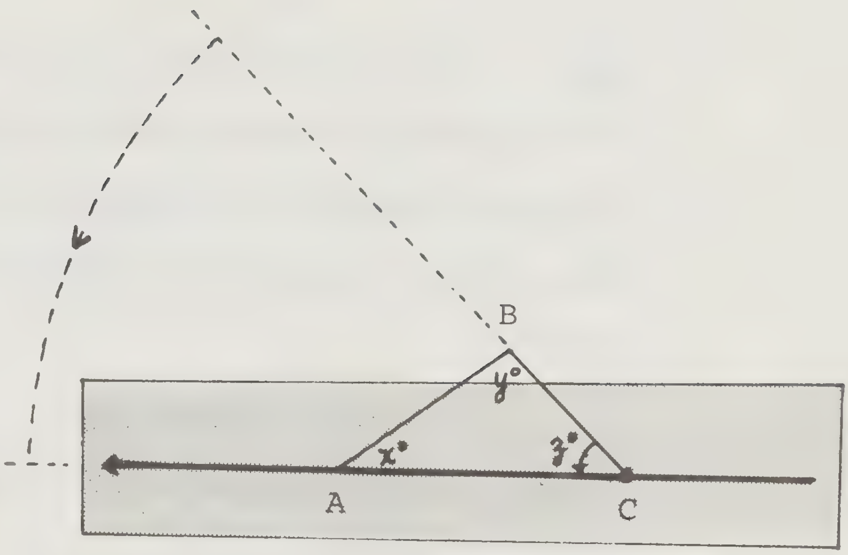
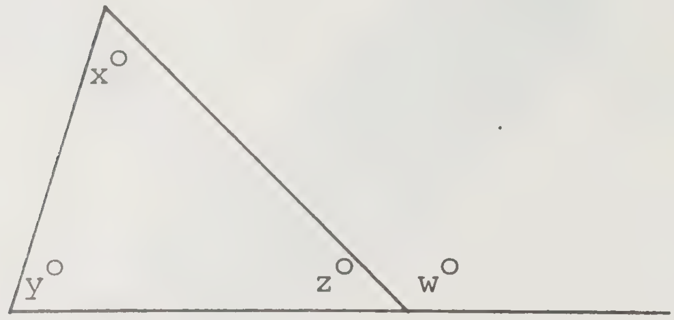


figure 4

Note: The above method can be used with quadrilaterals, pentagons, hexagons, and polygons in general — for both interior and exterior angles. From figure 1 to figure 3 above, the arrow has turned through the exterior angle at C and at the same time through $(x + y)^{\circ}$. These ideas can be demonstrated on an overhead projector, and then each student can use this technique individually in investigating angle-sum properties of a variety of polygons.

ii) <u>An exterior angle of a triangle equals the sum of the two interior-opposite angles</u>



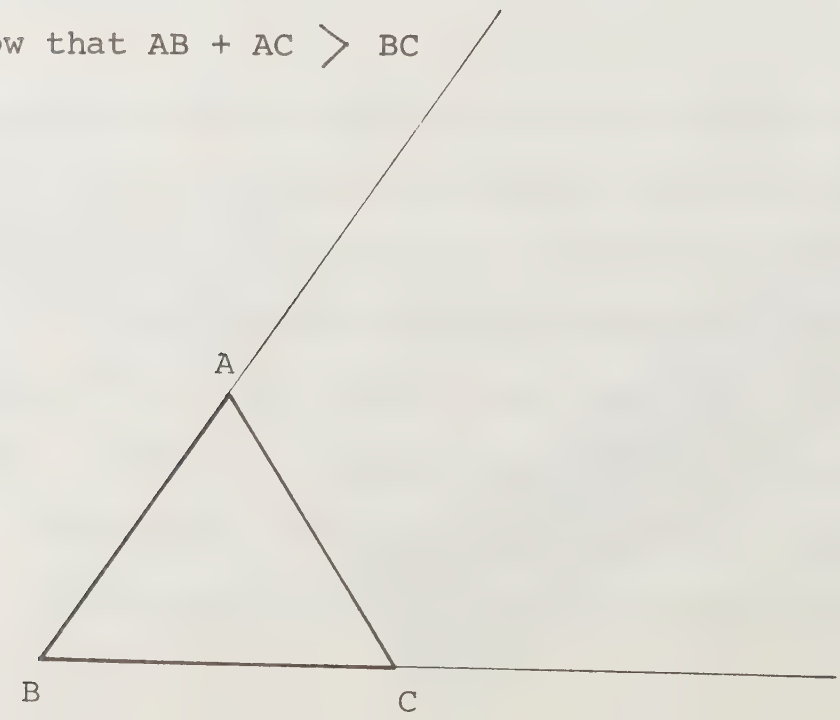
$w = x + y$

This property can be investigated by a slight modification of any of the above methods.

<u>Note</u>: The methods above for discovering the angle properties of a triangle can be modified to discover the angle properties of a quadrilateral and other polygons. These methods are only examples of the many ways by which these properties can be found.

iii) <u>The sum of two sides of a triangle is greater than the third side</u>

Show that $AB + AC > BC$



Extend BA and BC.

Using tracing paper, mark point C. Call it C_1.

With centre A turn C_1 onto BA extended.

$(BC_1 = BA + AC)$

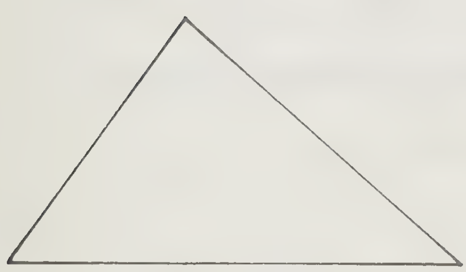
With centre B turn C_1 onto line BC.

(This shows $BC_1 > BC$; i.e. $BA + AC > AC$)

This property can also be demonstrated with:

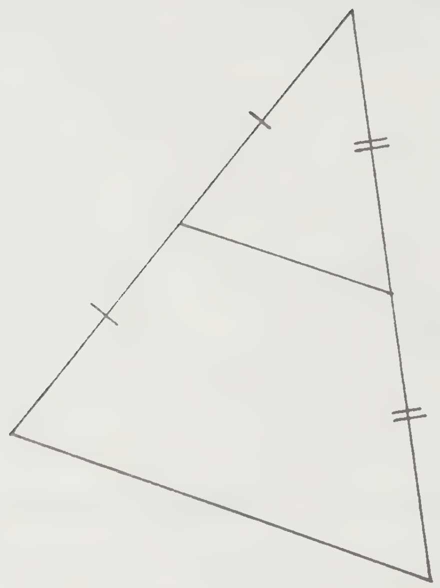
- a transparent mirror (Reflect AC onto BA produced. Mark C_1. Reflect BC_1 onto BC produced. Mark C_2. Observe $BC_2 > BC$; $\therefore BC_1 > BC$; $\therefore BA + AC > BC$.)
- a compass

iv) In a scalene triangle, angles and sides are unequal — <u>for any two angles, the side opposite the larger angle is longer than the side opposite the smaller angle</u>, and conversely.



This property can be investigated with the use of tracing paper, a transparent mirror, or a protractor and ruler.

v) The <u>segment joining the mid points of two sides of a triangle is parallel to the third side and equal to one half of it</u>.

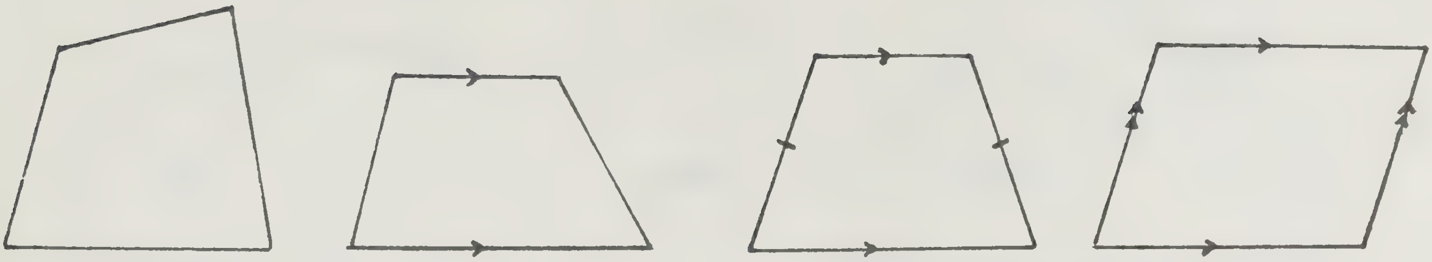


The figure can be accurately constructed, and then the property tested using tracing paper. This property is evident in tiling patterns formed by triangles. See the notes for 7G 3, pages 29 and 32. This property is a simple example of a dilatation with centre A and scale factor 2, and could be discussed after the idea of dilatation has been introduced.

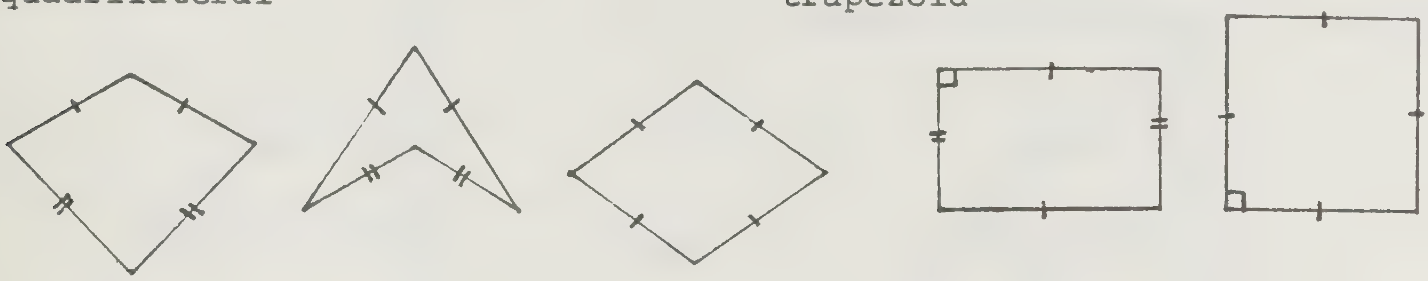
Quadrilaterals

Refer to the notes in 7G 1d, pages 20 to 26, dealing with the classification of quadrilaterals by side-angle relations and by symmetry, and in 7G 3ab, pages 4 to 20, 28 to 30, 34 and 35, dealing with properties of quadrilaterals developed from patterns with tiles. These properties can also be found or illustrated by measurement, by the use of tracing paper or a transparent mirror, or by the use of geoboards or dot paper (grids) for special cases of the quadrilaterals.

The figures below illustrate a way in which the different quadrilaterals may be defined using side-angle relations. Students could be asked to accurately construct examples of these figures, then to search for their other properties (by measurement, or by using tracing paper, or a transparent mirror).

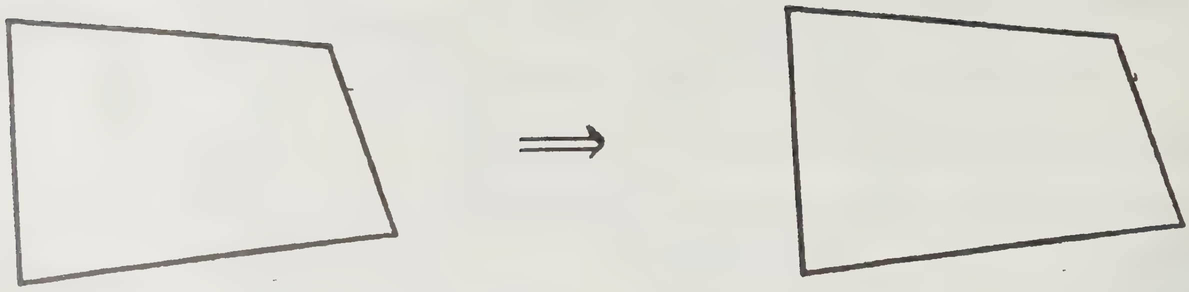


scalene quadrilateral | trapezoid | isosceles trapezoid | parallelogram



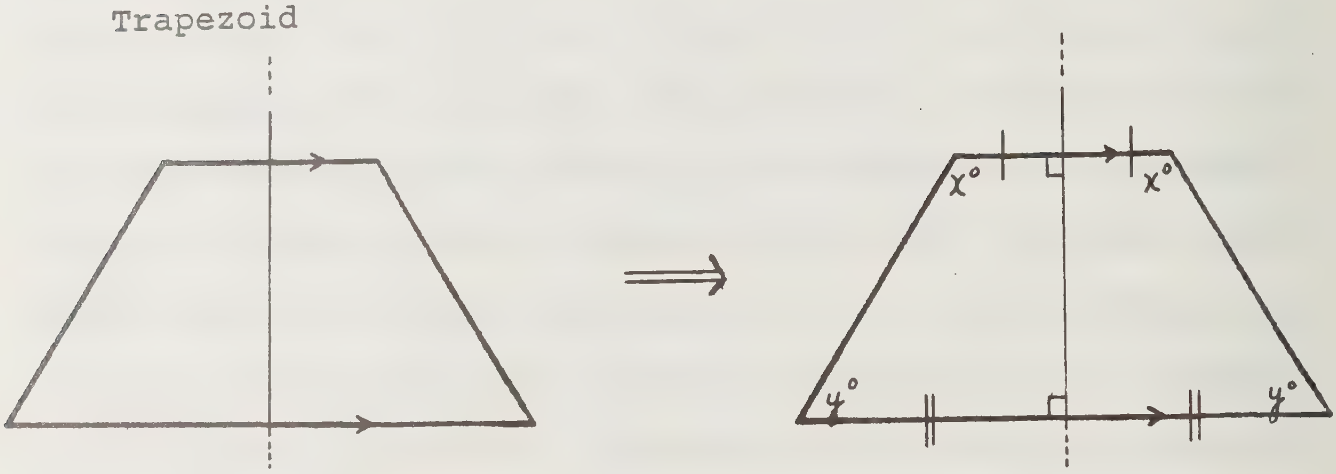
kite | chevron (kite) | rhombus | rectangle | square

The figures below illustrate the way different quadrilaterals can be defined in terms of their symmetry. The symmetries immediately reveal the corresponding congruent parts. For line-symmetry, find the corresponding parts visually using a transparent mirror or by tracing the figure then flipping the tracing. For rotational-symmetry, using tracing paper and tracing. For rotational-symmetry, use tracing paper and turn the figure about the centre of symmetry.



Scalene Quadrilateral

Trapezoid



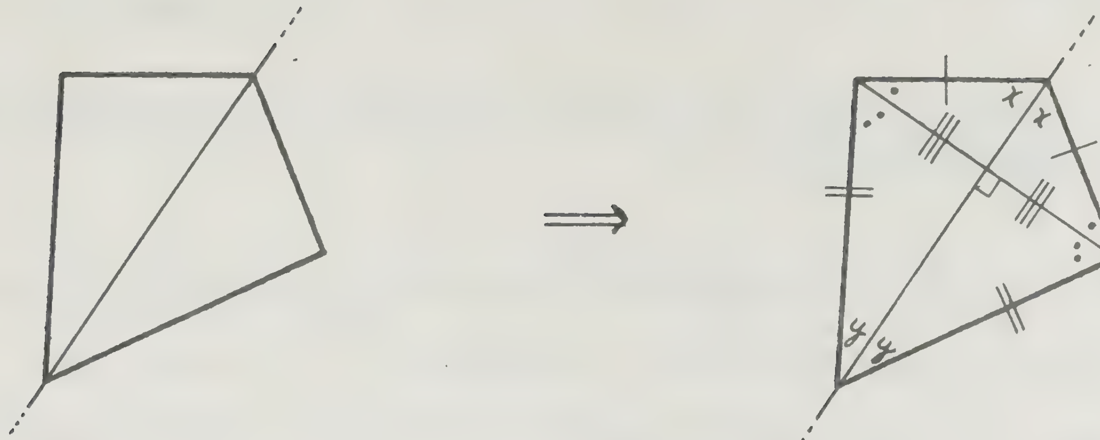
Isosceles Trapezoid

1 mid-points line of

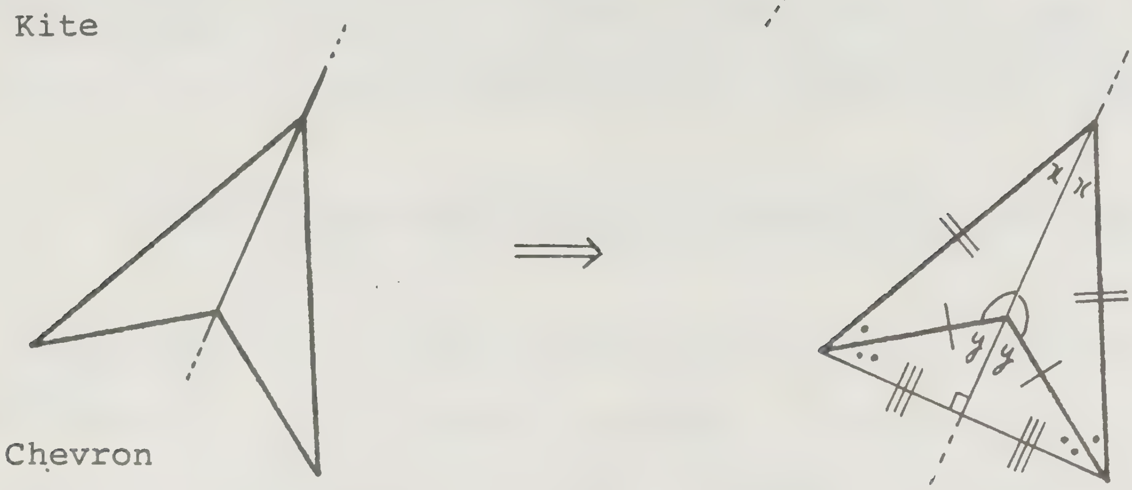
symmetry

Parallelogram

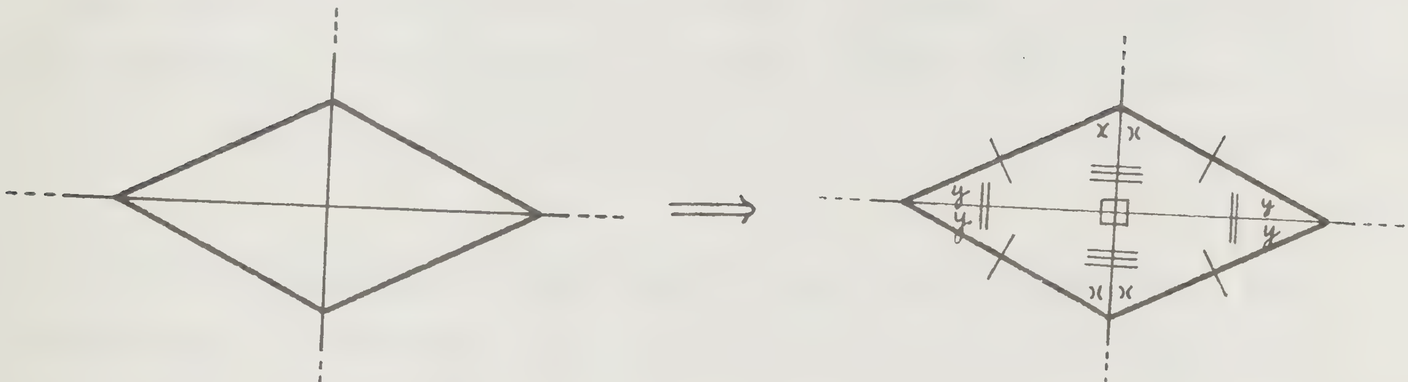
half-turn symmetry



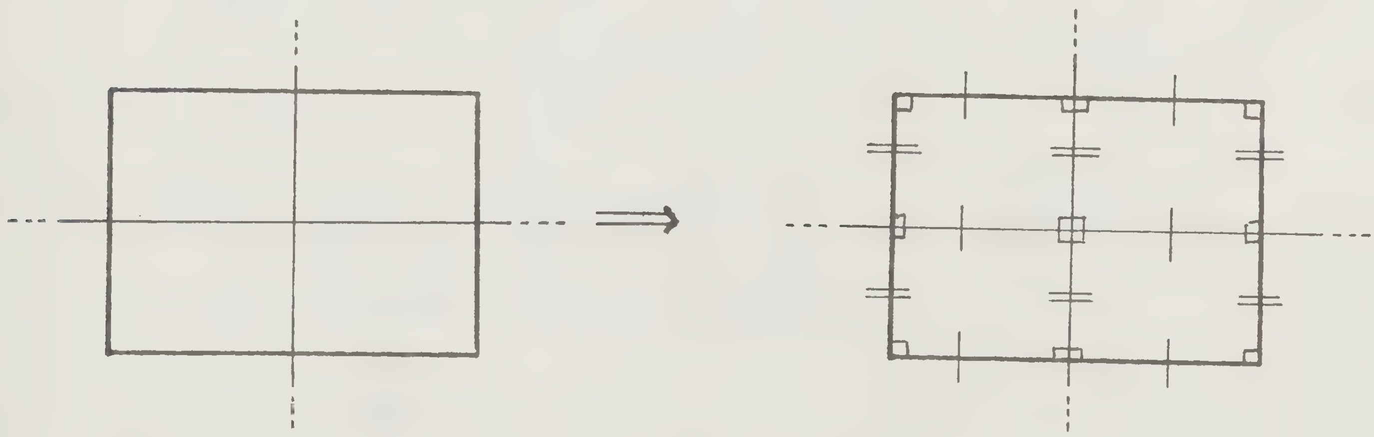
Kite



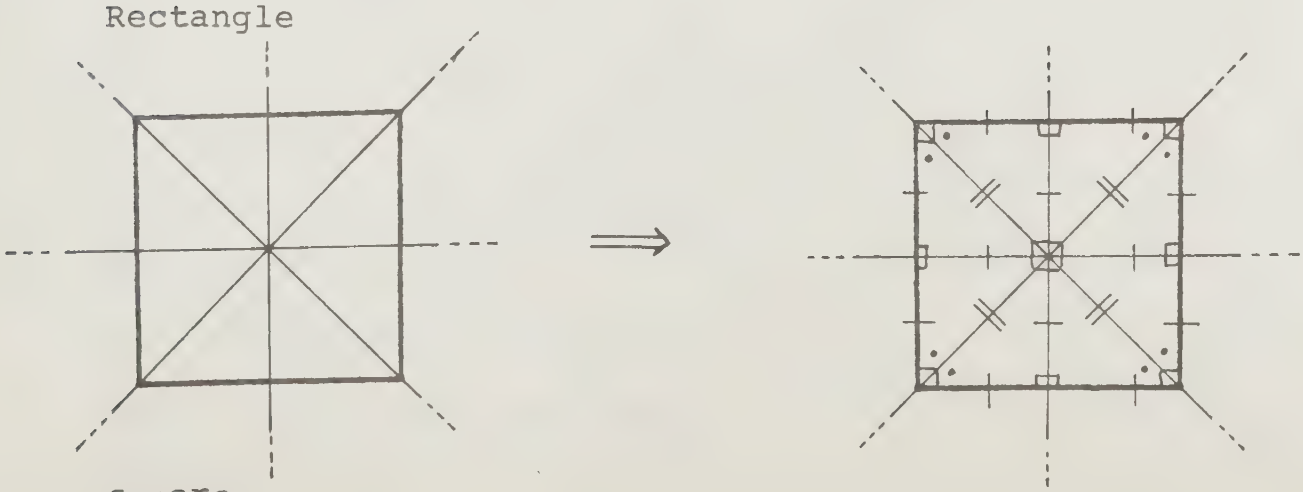
Chevron



Rhombus



Rectangle



Square

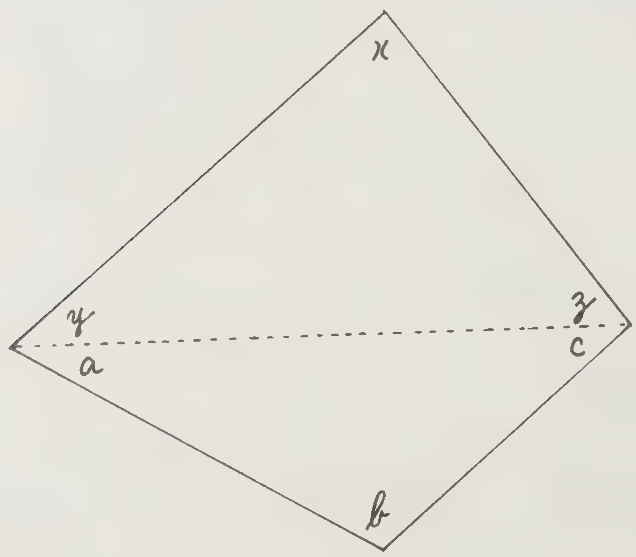
Scalene Quadrilaterals

Consider scalene quadrilaterals; they do not have equal sides, equal angles, parallel sides, or symmetry. The properties developed for these general quadrilaterals will hold for each of the special types of quadrilaterals.

i) The sum of the angles of a quadrilateral is 360^0

Refer to the notes dealing with methods for finding the sum of the angles of a triangle (pages 5 to 9). In particular, the method of drawing an arrow on tracing paper and turning it successively at each vertex is effective for investigating this property. See also page 9 in the notes for 7G 3.

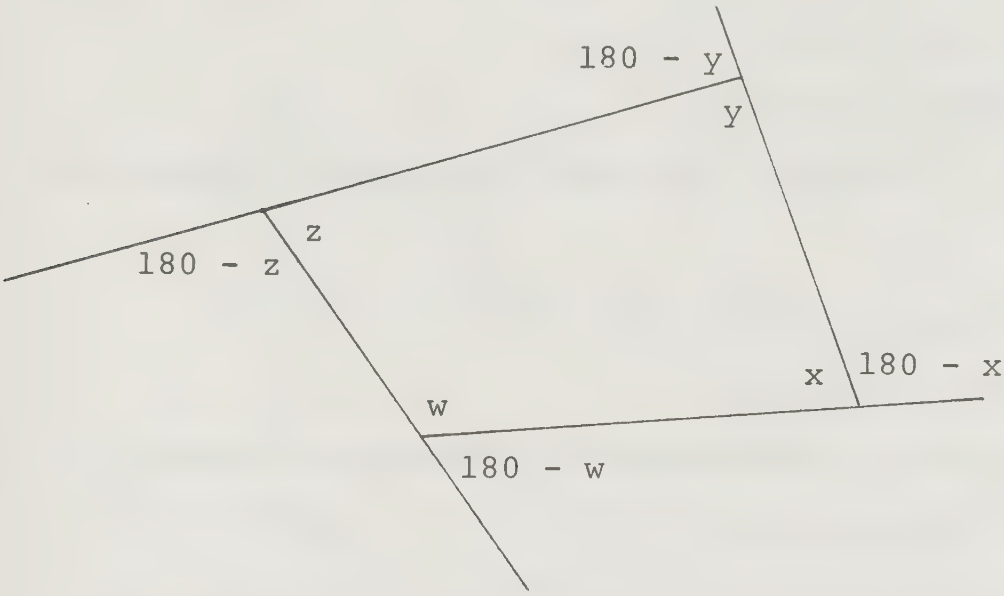
The technique of drawing a diagonal of the quadrilateral and then summing the angles of the two triangles is worthwhile.



ii) The sum of the exterior angles of a quadrilateral is 360°

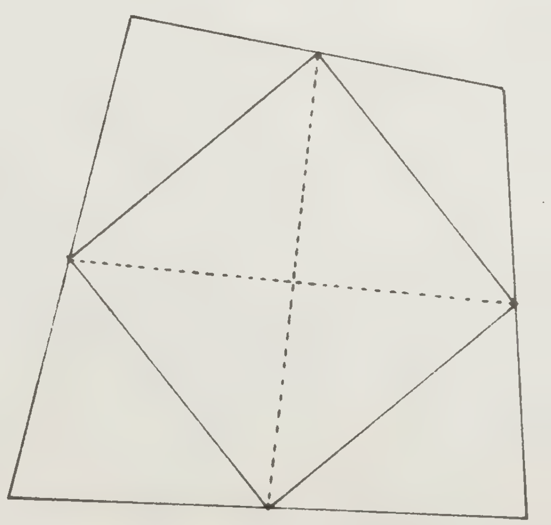
This property can be investigated in the same ways as property i) above.

The algebraic method is simple and gives the students a taste of how algebraic models can be used to prove results.



$$(180 - x) + (180 - y) + (180 - z) + (180 - w)$$
$$= 720 - (x + y + z + w)$$
$$= 720 - 360$$
$$= 360$$

iii) The figure formed by joining the mid points of the sides of a quadrilateral is a parallelogram



- Draw any quadrilateral.
- Accurately locate the mid points of each side.
- Join the mid points to produce the figure.
- Draw its diagonals, trace the figure, test it for half-turn symmetry.

Properties related to cyclic quadrilaterals are suggested for study in 9 Adv. G lb, 9 Gen. G lcd, and 10 Gen. G la.

Parallel Lines and a Transversal

The student should investigate properties of parallel lines related to:

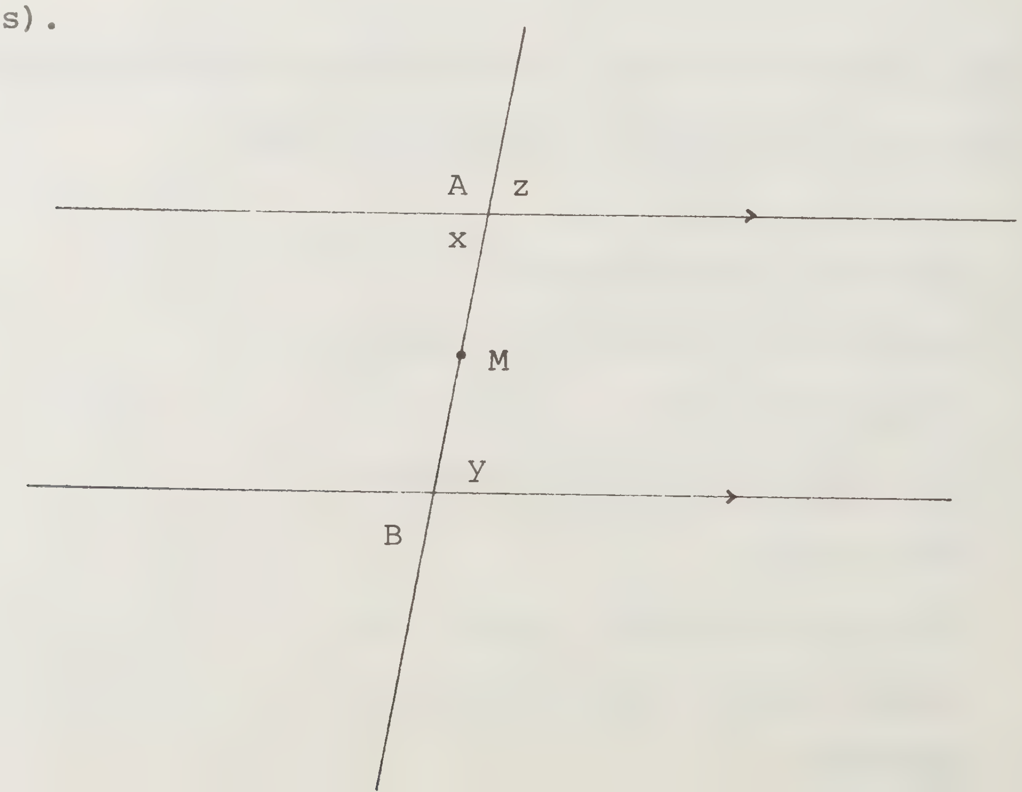
i) alternate angles

ii) corresponding angles

iii) interior angles on the same side of the transversal

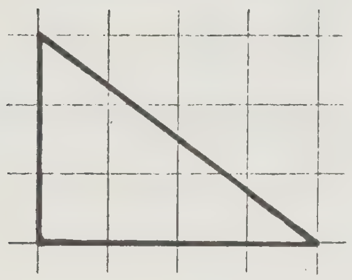
These properties may be discovered

- by measuring the angles;
- by investigating tiling patterns formed by triangles (notes for 7G 3 page 29);
- by tracing the figure, then half-turning it about the mid point of the segment of the transversal between the lines (alternate angles);
- by tracing the figure, then sliding it along the transversal until one parallel line fits onto the other (corresponding angles).



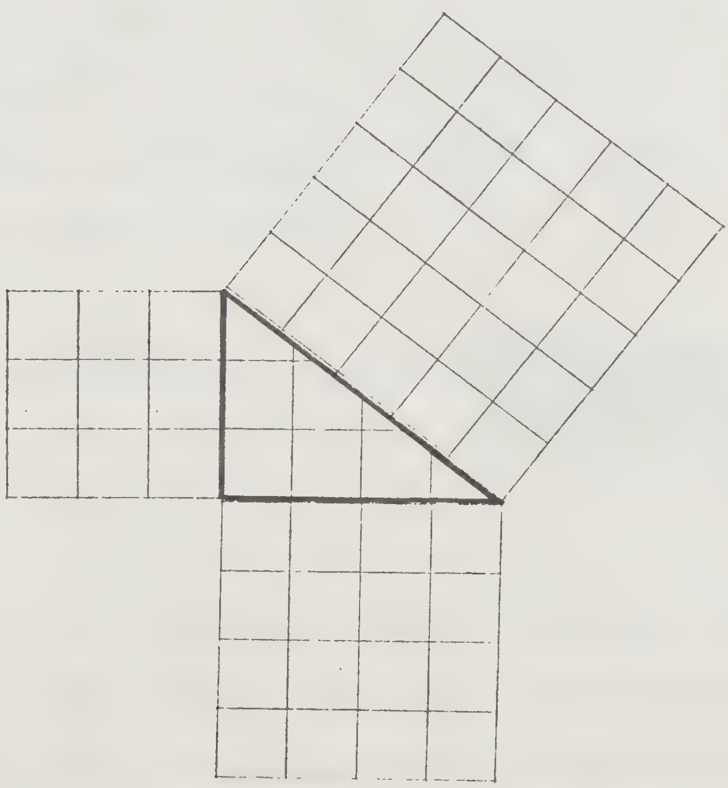
Pythagorean Theorem

Using square grid paper, construct right-angled triangles with legs 3 and 4, 6 and 8, 5 and 12. Measure the length of the hypotenuse of each triangle using the grid paper.



In this case, the length of the hypotenuse is 5 units.

Now cut out squares from the grid paper to fit on the three sides of each triangle, as illustrated below for the 3, 4, 5 triangle.

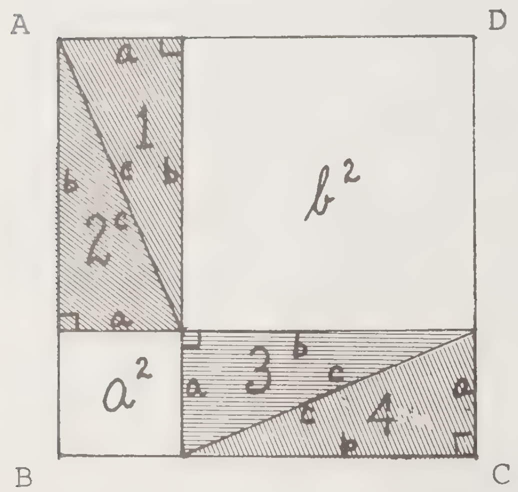


In each case, investigate the relation of the largest square to the other two.

Once the students have 'discovered' the relation for each of the triangles, it is reasonable to ask if this holds for all triangles.

There are numerous ways of demonstrating this. One of these is to have each student construct two or three right-angled triangles, measure their lengths, square these numbers, and test the results. This is a good activity, in which students measure, calculate and compare, and then have to recognize that their results correspond approximately to the 'theorem' under investigation.

Now the stage is set for a more logical discussion. Consider the large square ABCD below. It contains four congruent triangles.



These are shaded and labelled 1, 2, 3, and 4. It also contains two squares. These are <u>not</u> shaded. The area of the unshaded region is $a^2 + b^2$.

Now i) slide triangle 1 so that it fits into the corner at D;

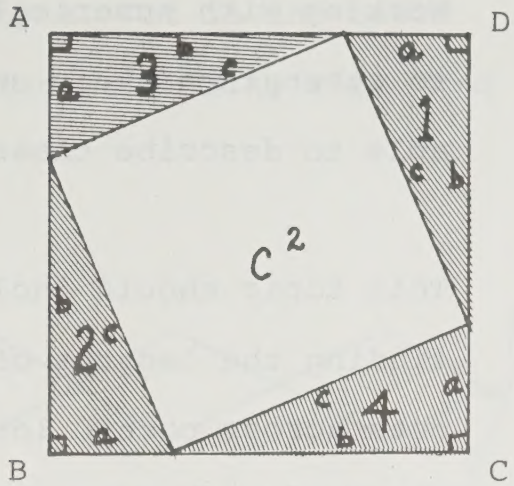
ii) slide triangle 2 so that it fits into the corner at B;

iii) slide triangle 3 so that it fits into the corner at A;

iv) leave triangle 4 in its present position.

(The above can be demonstrated on an overhead projector, using tiles.)

The figure on the right is the result.



The unshaded region is square. Its area is c^2. The areas of triangles 1, 2, 3, and 4 have not changed, nor has the area of square ABCD. Thus the area of unshaded region in the second figure is the same as in the first figure; that is $c^2 = a^2 + b^2$. Note that a and b are legs of the right-angled triangles, c is the hypotenuse. The theorem has been demonstrated for any right-angled triangle. This theorem should be related to real-life situations and numerous simple problems should be solved for numerical cases as suggested in topic b) below.

b) <u>Applying the properties of a) in numerical exercises</u>

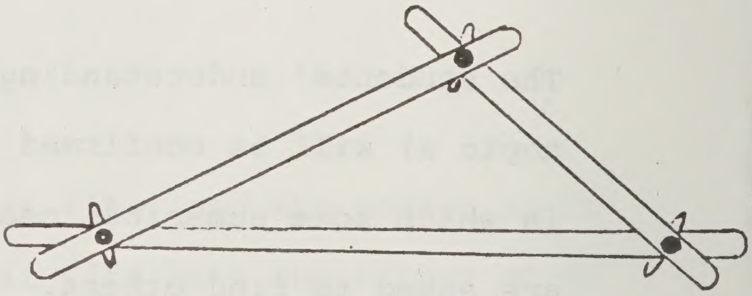
The students' understanding of the properties developed in topic a) will be confirmed and expanded by solving problems in which some numerical measures are provided and the students are asked to find others. Many students are able to find correct numerical solutions, even though they experience difficulty in expressing the properties accurately in words or symbols.

Working with numerical examples of the properties will help to strengthen their understanding; eventually, they should be able to describe these properties in more general terms.

This topic should include real-world problems such as finding the lengths of guy wires, supporting struts, short cuts across corner lots, and so on. At this stage, however, the student has not been formally introduced to ways of finding square roots of numbers. The student will likely need to find approximate answers by a systematic 'guess and test' procedure. The use of a calculator will greatly facilitate the ability to approximate. The introduction of Newton's Method would seem to be premature at this grade level.

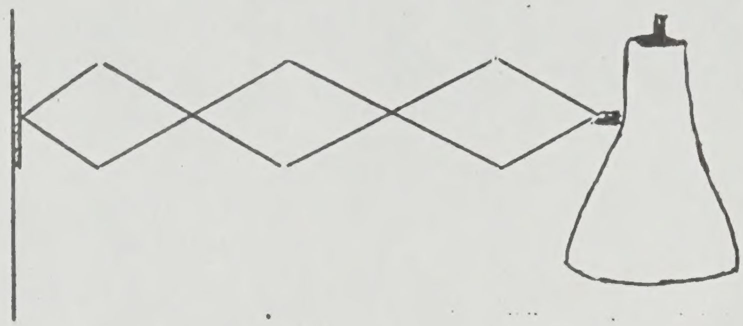
c) Rigidity and non-rigidity of polygonal figures; real life applications

Students should investigate the rigidity of various polygonal figures made from rigid strips of bristol board joined together by paper fasteners. The discovery that only the triangle is rigid is of fundamental importance in construction and engineering trades.



A trianglular figure is a rigid structure. It is the basic unit in the construction of frame bridges, drilling rigs, and scaffolding.

Also of significance are non-rigid figures such as the pantograph which traces an enlargement of an original diagram, and the expanding rhombus which is used in moveable light fixtures.



The book Machines, Mechanisms and Mathematics, available from Clark, Irwin & Co. Ltd., has many interesting investigations of linkages.

d) Polygons; properties of regular polygons; symmetry

See the notes for 7G 1e for a discussion of this topic. The students might investigate the polygon for properties related to the sum of the interior angles and the sum of the exterior angles. They can use methods similar to the ones suggested earlier for the angle sum of a triangle and of a quadrilateral. It is reassuring to find the same results by such a variety of methods. Properties of scalene polygons hold for regular polygons.

Students could be asked to investigate ways of accurately constructing regular polygons of 5, 6, 8, 10, and 12 sides. They could investigate the line- and rotational symmetries of these figures.